

# Proper Orbifold Cohomotopy for M-Theory

Urs Schreiber on joint work with Hisham Sati

NYU AD Science Division, Program of Mathematics

& Center for Quantum and Topological Systems

New York University, Abu Dhabi

talk at:

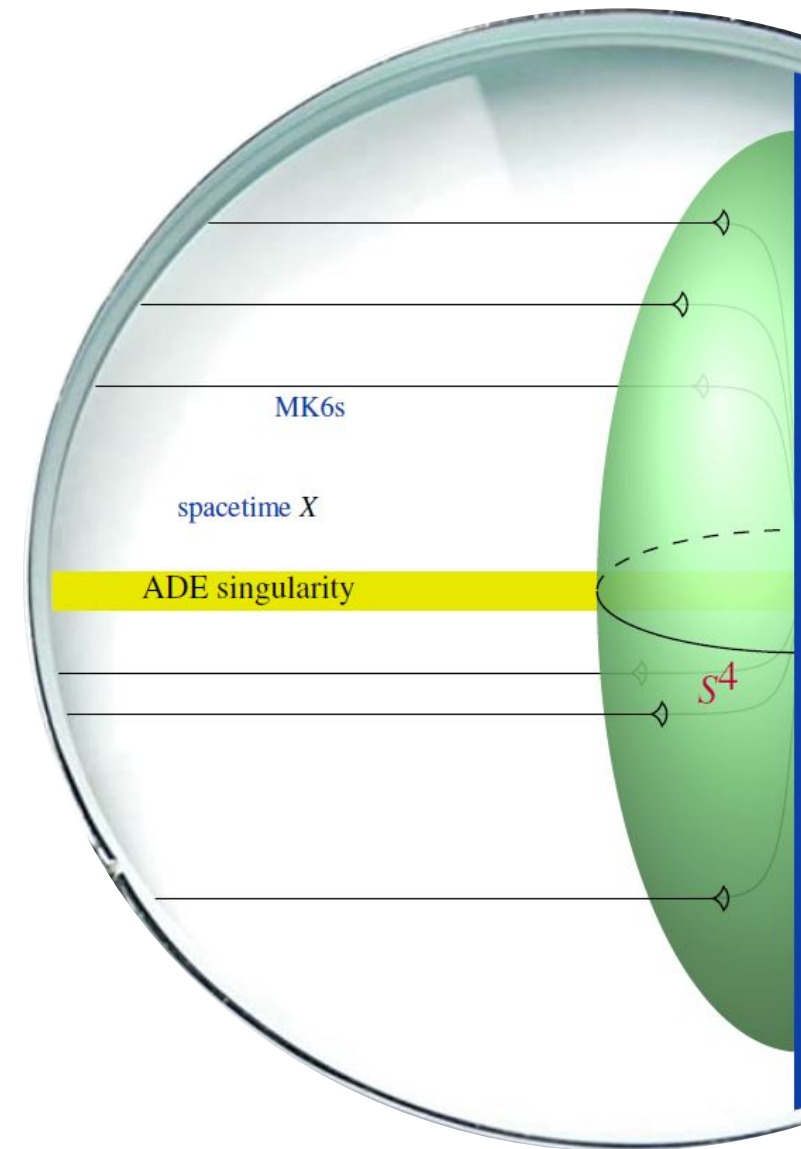
String and M-Theory

The New Geometry of the 21<sup>st</sup> Century

II

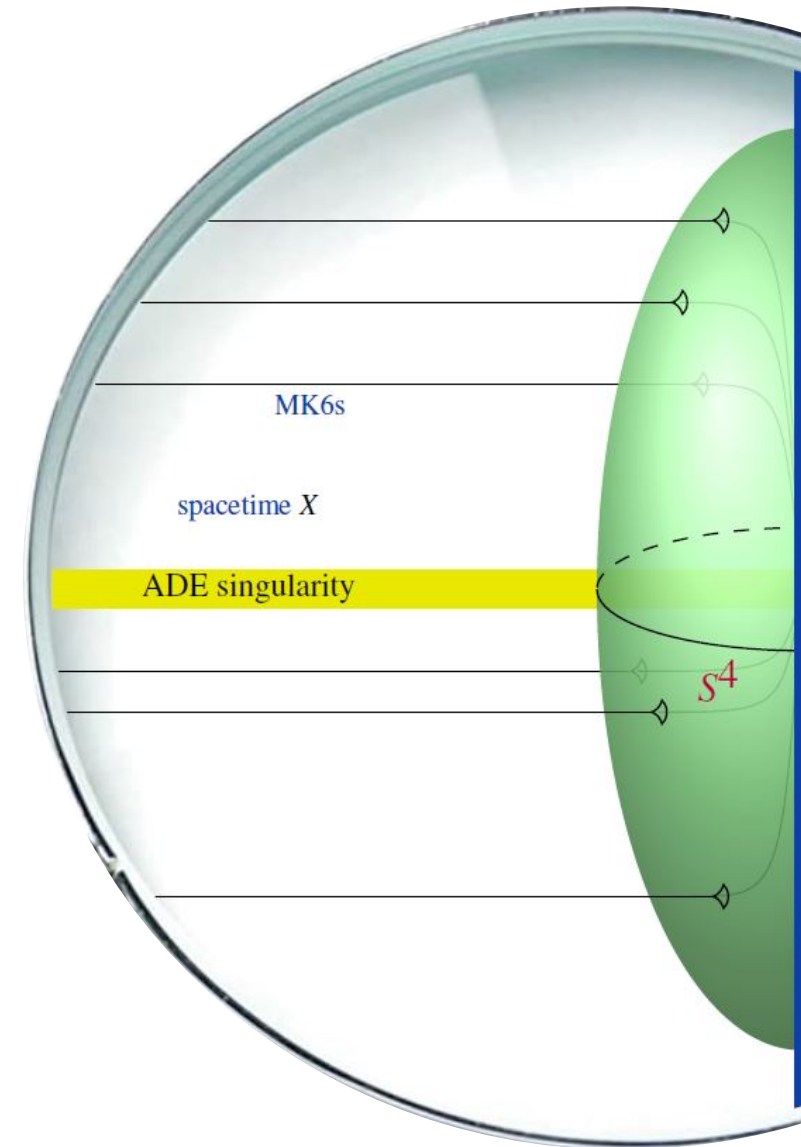
via NUS Singapore, Nov.-Dec. 2021

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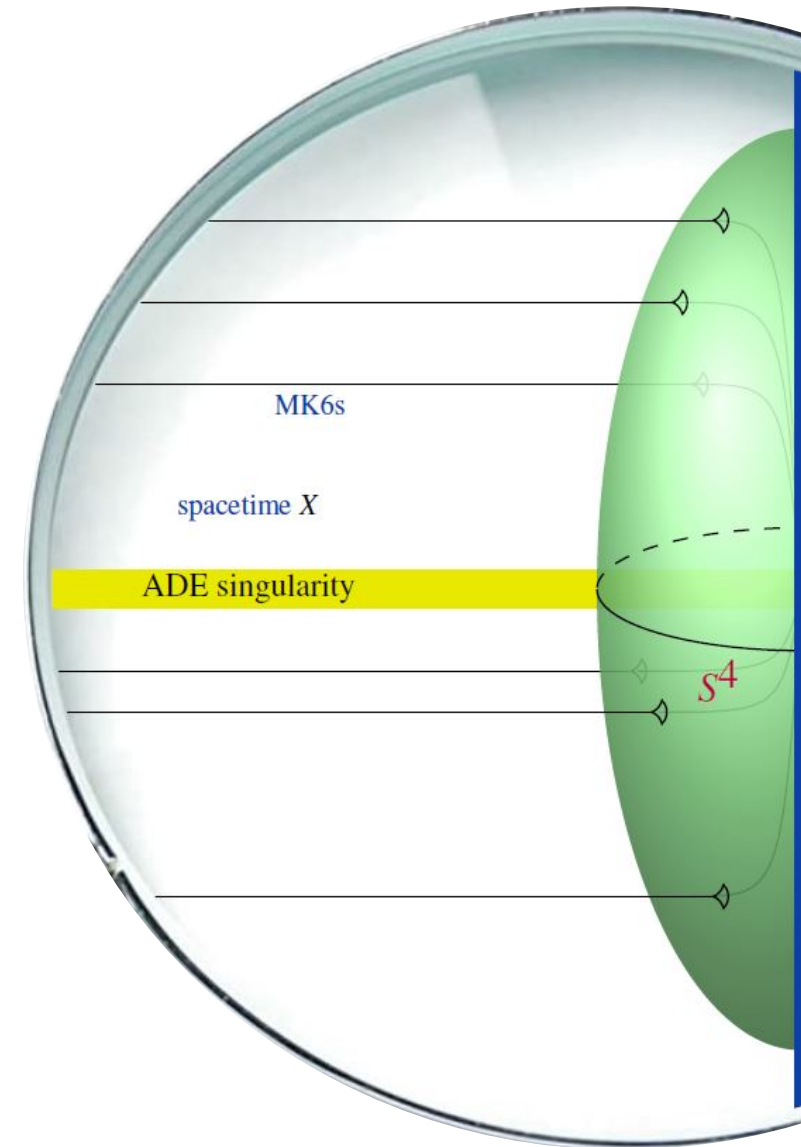


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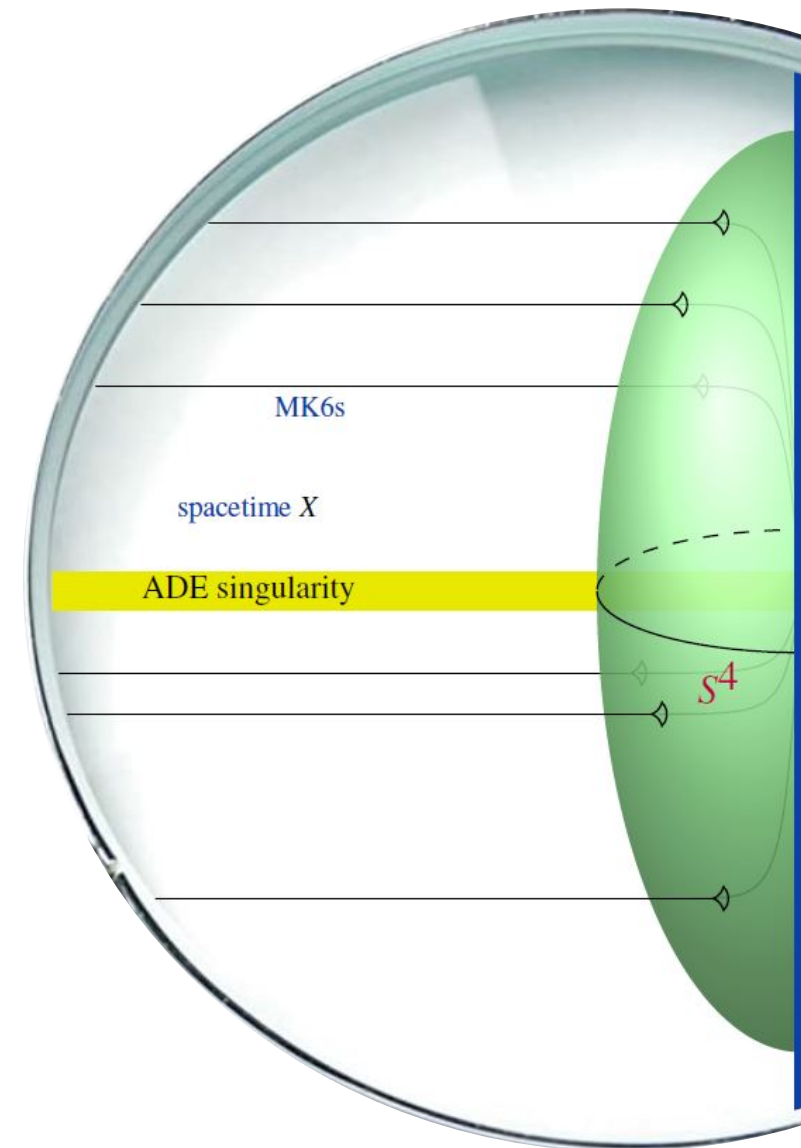
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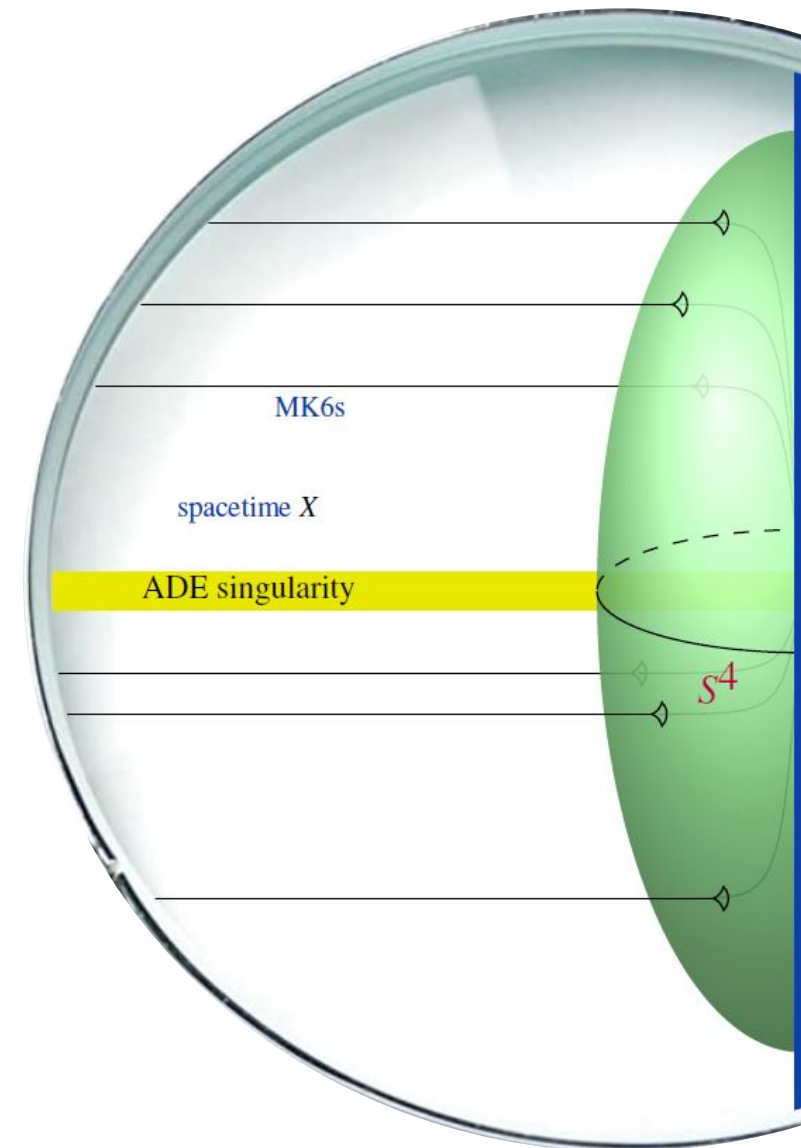
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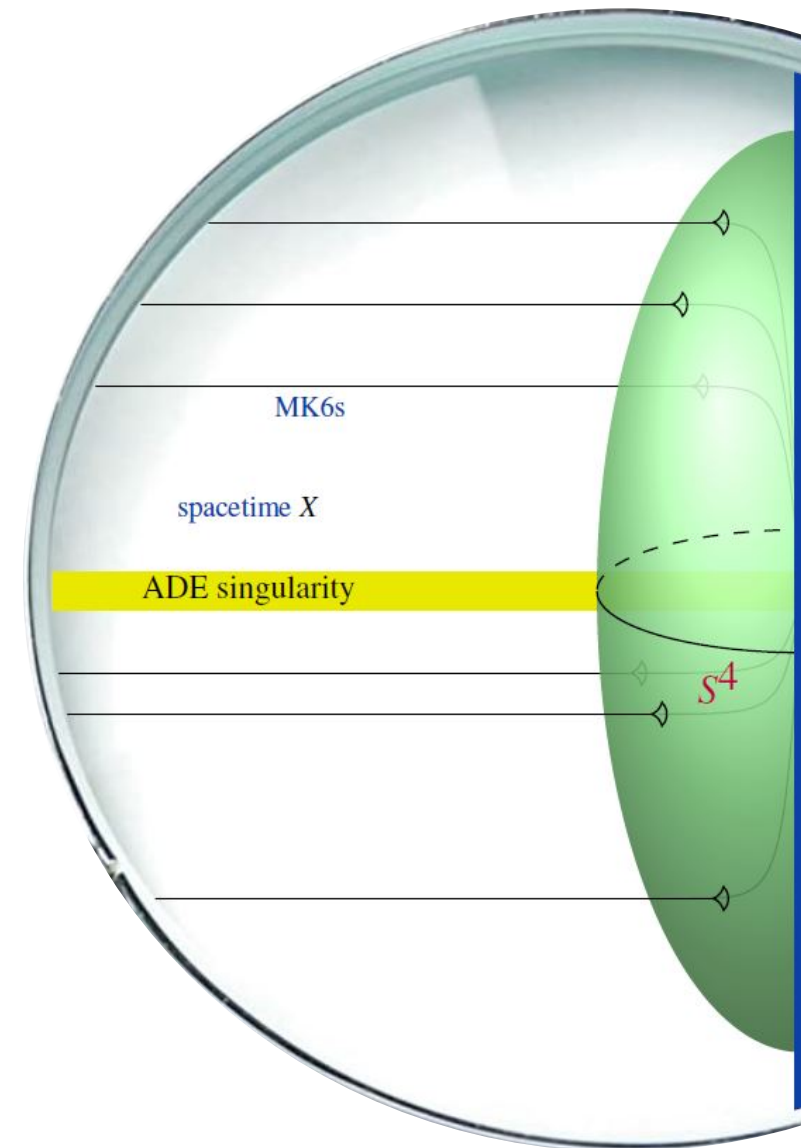
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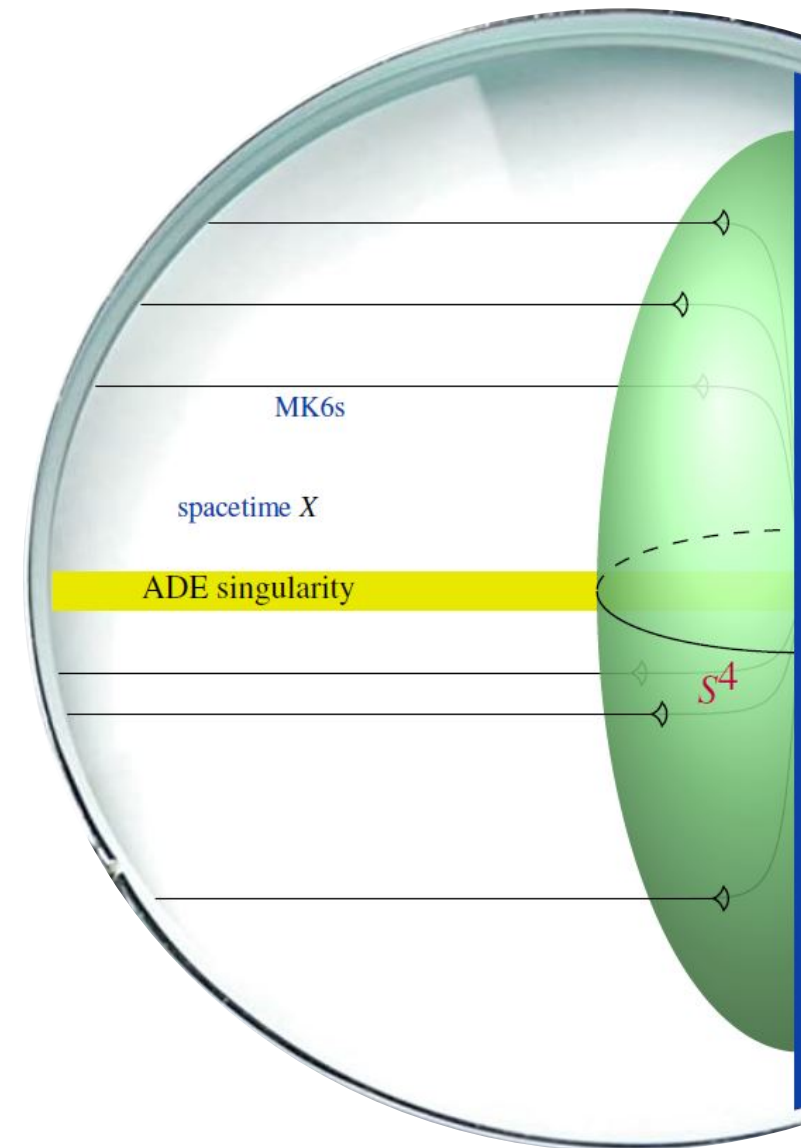
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For more see H. Sati's talks and see: [ncatlab.org/schreiber/show/Hypothesis+H](https://ncatlab.org/schreiber/show/Hypothesis+H)





higher orbi-geometry

orbifold cohomology

I – Proper Orbifold Cohomotopy

II – for M-Theory

Hypothesis H

M5-brane charges in  
flat orbi-orientifolds

*String theory at its finest is, or should be, a new branch of geometry  
... developed in the 21st century ... that fell by chance into the 20th century ...*

*To elucidate the proper generalization of geometry  
[is] the central problem of string theory.*

E. Witten (1988)

as quoted on p. 95, 102 in:

P C W Davis and J Brown (eds.)

*Superstrings: A theory of everything?*

Camb Univ Press 1988, 1991: Canto 1992

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Lots of subtleties, but two general principles:

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Ex.: 3d CS theory with cpt gauge group:  
 [Hit90] [APW91]

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Ex.: D6  $\perp$  D8-branes via Hypothesis H  
 [SS19-Quant][CSS21-Quant]

---

$\mathcal{X}$  = transverse cptfd. space to branes  
 $\mathcal{A}$  = moduli stack of diff. 4-Cohomotopy  
 $\widehat{H}_{\text{pos}}^{\bullet}$  = positive ordinary cohomology

The dictionary:

**physics**

**mathematics**

geometry

topos theory

+ gauge principle

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$\infty$ -topos theory [Si99][Lu03,09][TV05][Re10]

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The jargon:

$$\mathcal{X}, \mathcal{A} \in \mathbf{H} \simeq \text{Sh}_\infty(\mathbf{S})$$

(examples follow)

$\infty$ -topos

$\infty$ -stacks

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21st geometry

high. geom. spaces

local model geom.

# The Higher Geometry of Physics.

[JSSW18-HigStrc][FSS19-RatM][SS20-OrbCoh]

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The key point:

**stringy geometry**

$\leftrightarrow$

**higher homotopy**

$p$ -brane charges

$\pi_{p+1}(\mathcal{A}) \in \text{Grp}(\mathbf{H}_0)$

$p_1 \perp p_2$ -intersections

higher k-invariants

[FSS13-Bouq, §3]

[HSS18-ADE, §2]

[FSS19-RatM, §7]

$$\Sigma \in \mathbf{S}, \quad \mathcal{X} \in \mathbf{H}$$

*representable* *probe worldvol.* *higher stack* *gauged target sp.*

The idea:

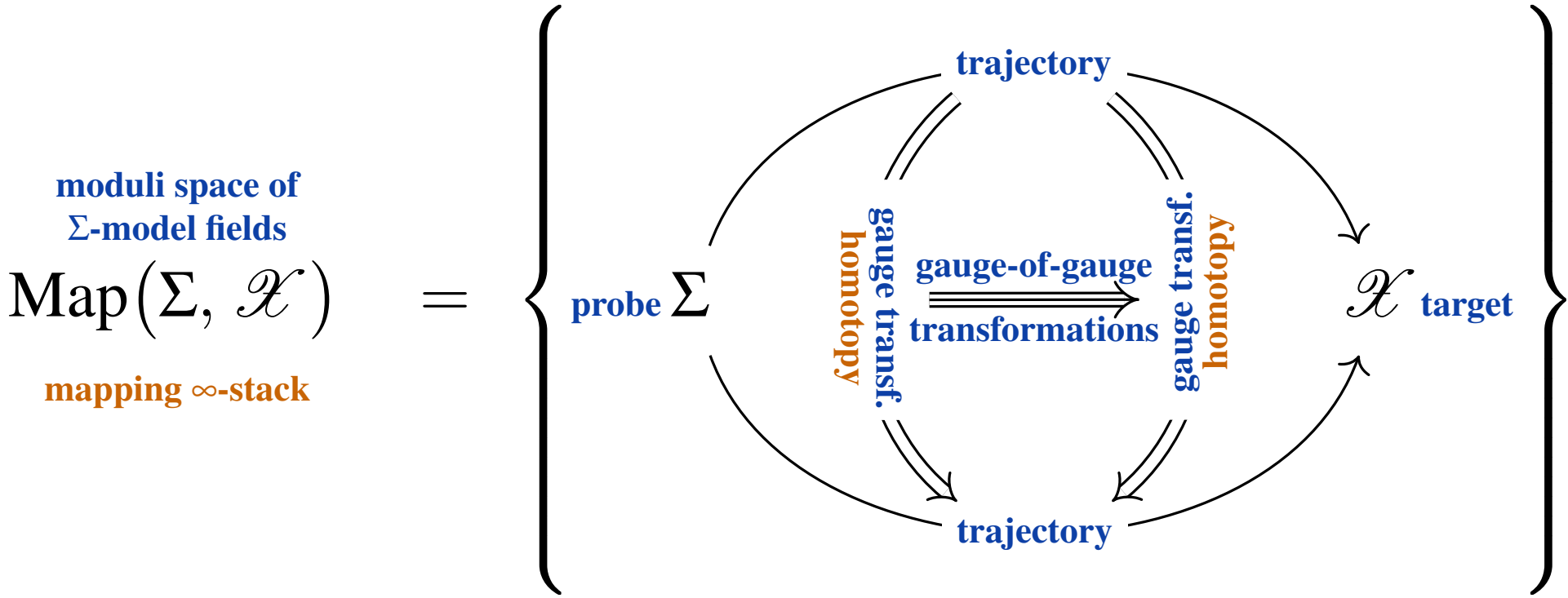
Just as an emergent target space  $\mathcal{X}$  seen via probes by worldvolumes  $\Sigma$ ,  
so an  $\infty$ -**stack**  $\mathcal{X}$  is a space bootstrapped by its gauged system of  $\Sigma$ -plots:

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Key example: Higher geometry locally modeled on  $\text{CartSp} = \{ \mathbb{R}^n \xrightarrow{\text{smooth}} \mathbb{R}^{n'} \}$ :

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faithfully subsumes all differential topology:

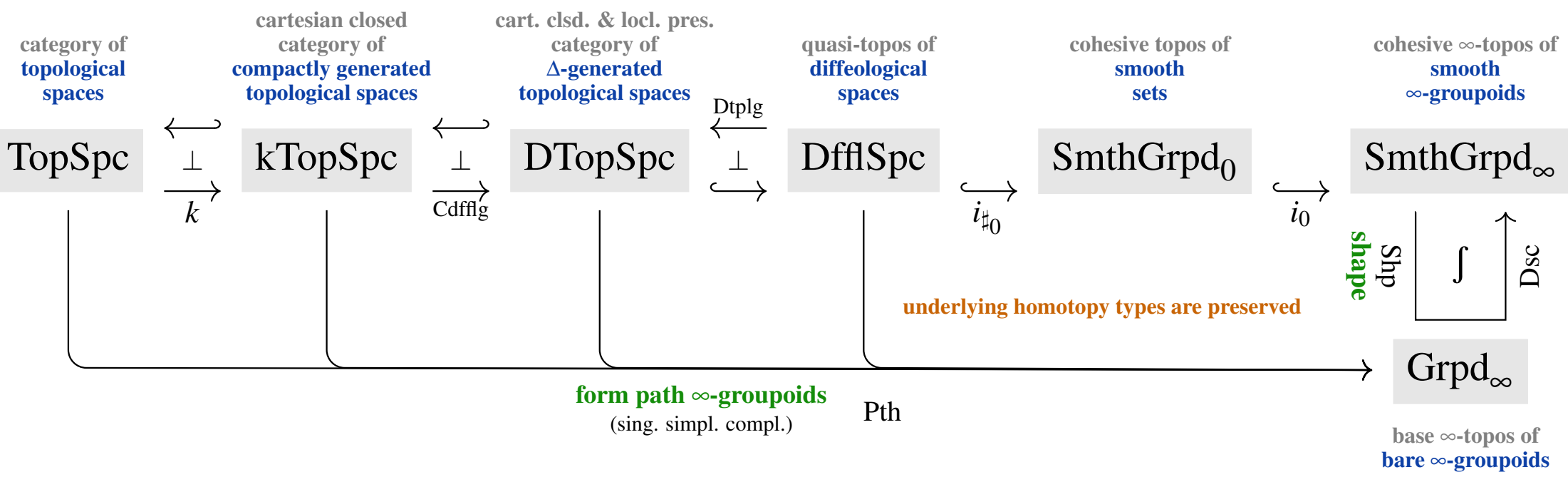
# Cohesive homotopy theory of Smooth $\infty$ -stacks.

[SSS09][Sc13][ScSh14][SS20-OrbCoh]

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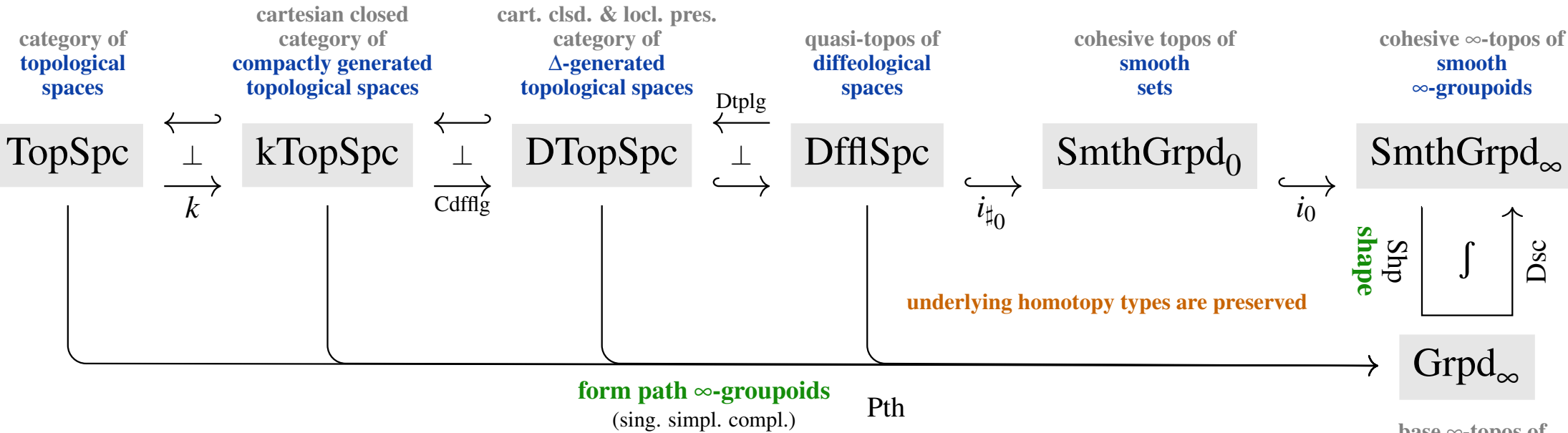
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but subsumes also moduli for all

**higher gauge fields**  $\leftrightarrow$  **differential cohomology** [FSS20-Char, §4.3]

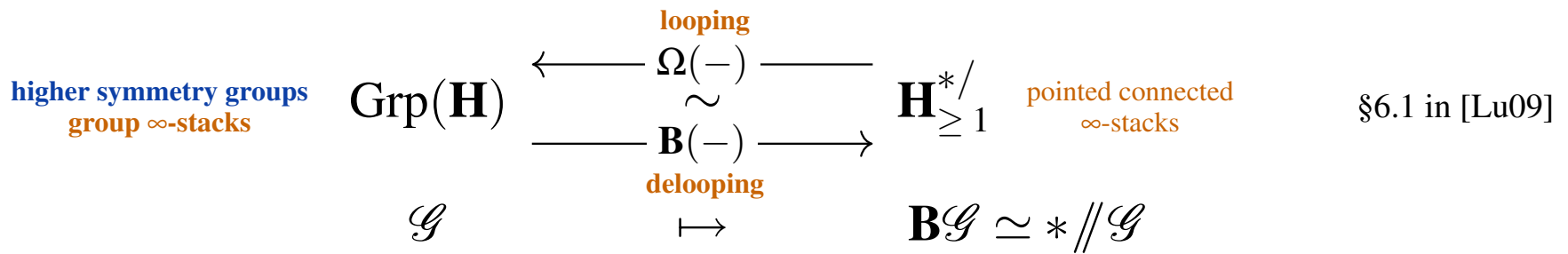
in particular for *abelian* higher gauge fields:

$$\text{Spectra}(\text{SmthGrpd}_\infty) = \left\{ \begin{array}{l} \text{abelian generalized differential} \\ \text{cohomology theories} \end{array} \right\} \quad \begin{array}{l} [\text{Sc13}] [\text{BNV14}], \\ \text{review in } [\text{ADH21}] \end{array}$$

# Higher symmetry.

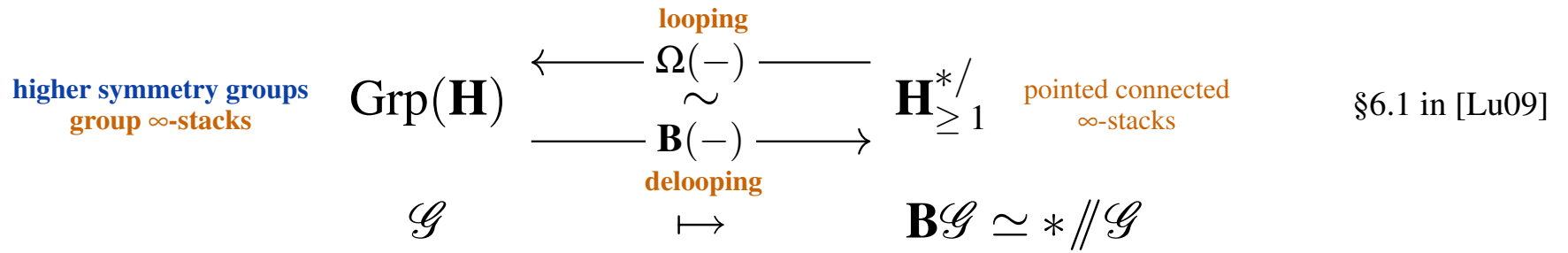
$\infty$ -Toposes  $\mathbf{H}$  know all about *higher symmetries* – i.e. *n-group symmetries* for  $n \in \mathbb{N} \sqcup \{\infty\}$ ):

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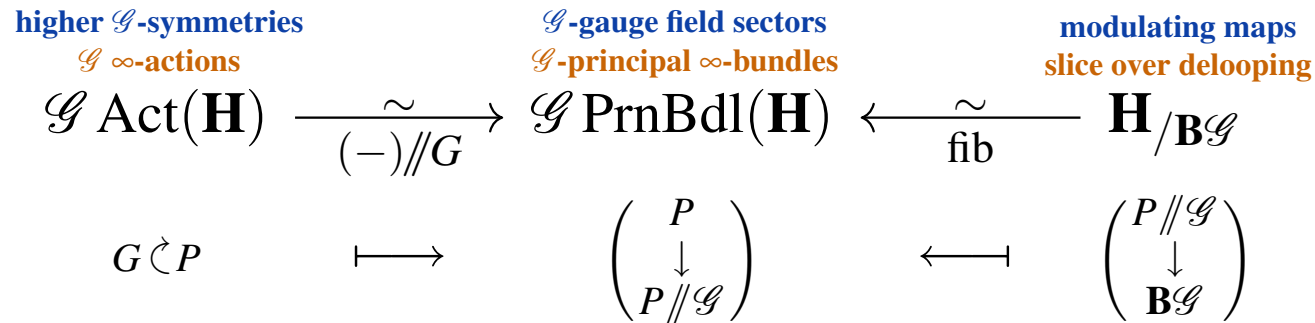


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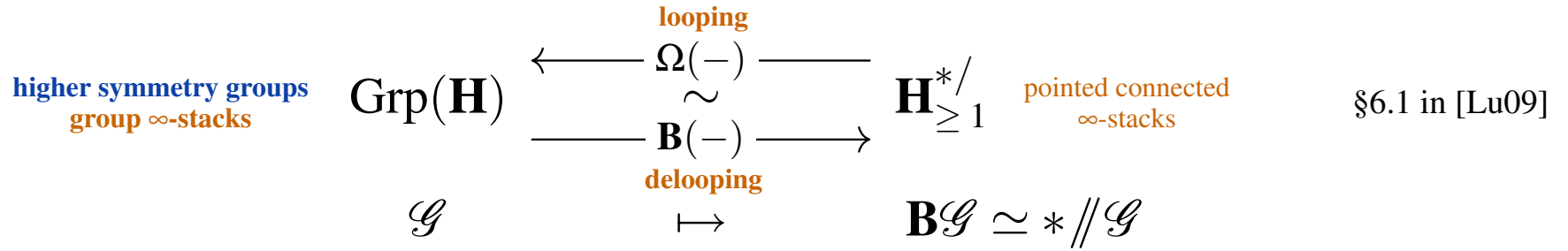
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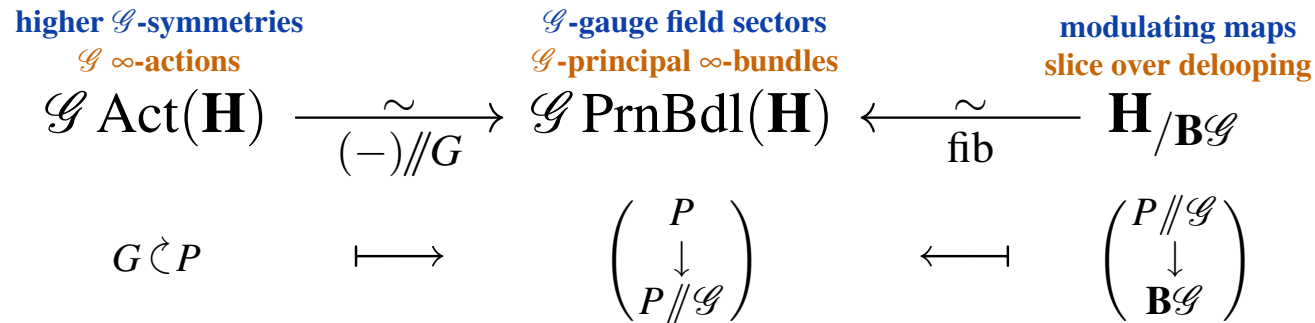
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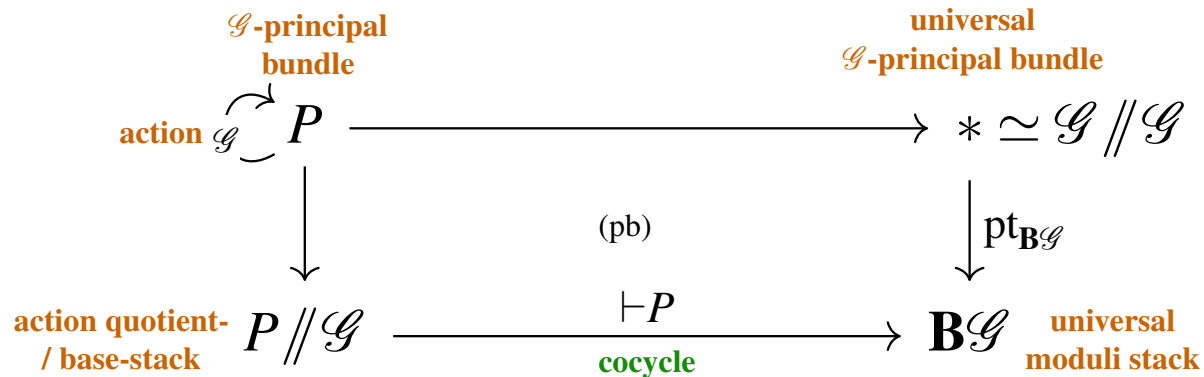
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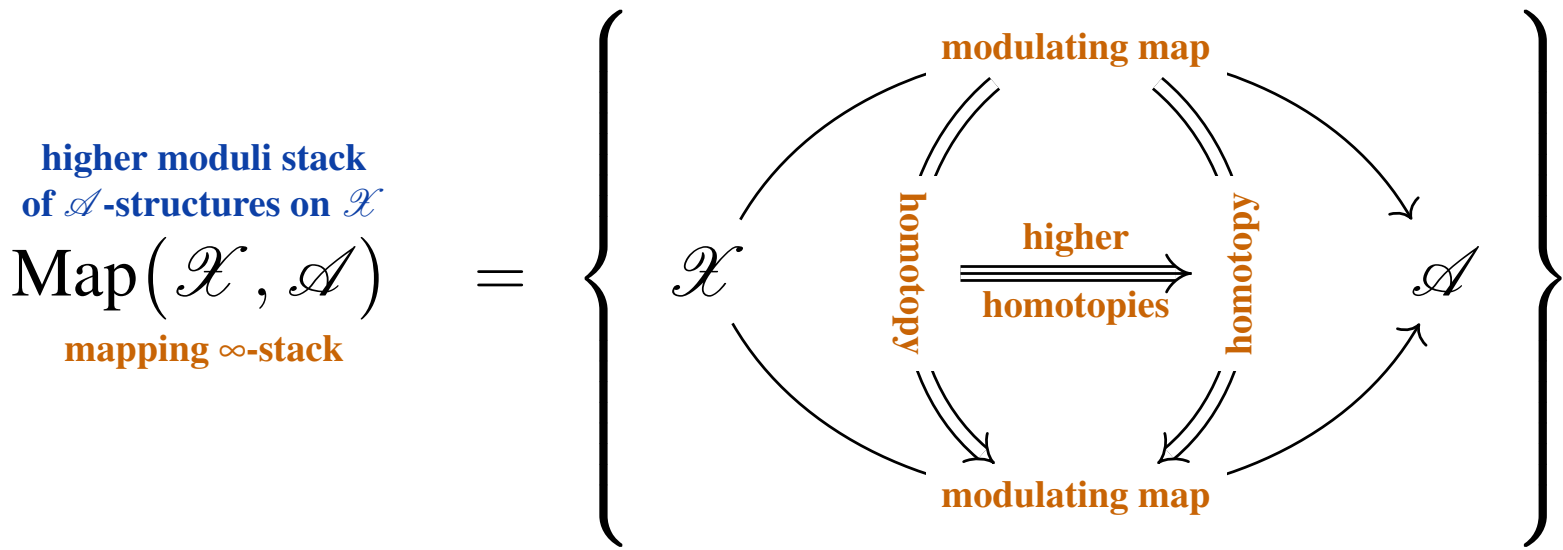
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Technical side remark. – The correspondence is enacted by homotopy cartesian squares of this form:



Maps *out* of an  $\infty$ -stack  $\mathcal{X} \rightarrow \mathcal{A}$  encode  $\mathcal{A}$ -moduli on  $\mathcal{X}$ :



Ex.:  $\text{Map}(\mathcal{X}, \mathbf{BG})$  is the moduli  $\infty$ -stack of  $\mathcal{G}$ -principal  $\infty$ -bundles on  $\mathcal{X}$  (high. gauge field sect.)



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$$\text{higher moduli stack of } \mathcal{A}\text{-structures on } \mathcal{X} \\ \text{mapping } \infty\text{-stack} \\ \text{Map}(\mathcal{X}, \mathcal{A}) = \left\{ \begin{array}{ccc} & \text{modulating map} & \\ \mathcal{X} & \begin{array}{c} \text{homotopy} \\ \text{higher} \\ \text{homotopies} \\ \text{homotopy} \end{array} & \mathcal{A} \\ & \text{modulating map} & \end{array} \right\}$$

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Fund. Thm. [Lu09]: for  $\mathcal{B} \in \mathbf{H}$  also  $\mathbf{H}_{/\mathcal{B}}$  is an  $\infty$ -topos, whose objects are maps  $\mathcal{X} \xrightarrow{c} \mathcal{B}$ , with

$$\text{slice mapping } \infty\text{-stack} \\ \text{Map}((\mathcal{X}, c), (\mathcal{A}, c'))_{\mathcal{B}} = \left\{ \begin{array}{ccc} & & \\ \mathcal{X} & \begin{array}{c} \text{homotopy} \\ \text{higher} \\ \text{homotopies} \\ \text{homotopy} \end{array} & \mathcal{A} \\ & \begin{array}{c} c \\ \text{homotopy} \\ c' \end{array} & \mathcal{B} \end{array} \right\}.$$

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**Higher symmetry – Example: GS-mechanism. [SSS09][FSS12][FSS20-M5Str][SS20-M5GS]**

Lie groups

diffeological groups

smooth  $\infty$ -groups

Under  $\text{Grp}(\text{SmthMfd}) \hookrightarrow \text{Grp}(\text{DfflSpc}) \xleftarrow{\text{Grp}(i_0, \#_0)} \text{Grp}(\text{SmthGrpd}_\infty)$

**mapping stack into delooping...**

**is moduli stack of:**

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$\text{Map}(X, \mathbf{B}\text{Spin}(n))$	Spin-bundles

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$\text{Map}((X, \mathbf{c}_1), (\mathbf{BSO}(n), \mathbf{w}_2))_{\mathbf{BU}(1)}$ $\simeq \text{Map}(X, \mathbf{BSpin}^{\mathbf{c}_1}(n))$	$\text{Spin}^c$ -bundles

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$\text{Map}((X, \mathbf{c}_1), (\mathbf{BSO}(n), \mathbf{w}_2))_{\mathbf{BU}(1)}$ $\simeq \text{Map}(X, \mathbf{BSpin}^{\mathbf{c}_1}(n))$	Spin <sup>c</sup> -bundles
$\text{Map}((X, 0), (\mathbf{BSpin}(n), \frac{1}{2}p_1))_{\mathbf{B}^3\mathbf{U}(1)}$ $\simeq \text{Map}(X, \mathbf{BString}(n))$	String 2-bundles

**Higher symmetry – Example: GS-mechanism.** [SSS09][FSS12][FSS20-M5Str][SS20-M5GS]

Lie groups                      diffeological groups                      smooth  $\infty$ -groups

$$\text{Under } \text{Grp}(\text{SmthMfd}) \hookrightarrow \text{Grp}(\text{DfflSpc}) \xleftarrow{\text{Grp}(i_0, \#_0)} \text{Grp}(\text{SmthGrpd}_\infty)$$

**mapping stack into delooping...**

**is moduli stack of:**

$\text{Map}(X, \mathbf{BU}(1))$	circle bundles
$\text{Map}(X, \mathbf{B}^{p+1}\mathbf{U}(1))$	bundle $p$ -gerbes
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$\text{Map}((X, \mathbf{c}_2), (\mathbf{BSpin}(n), \frac{1}{2}\mathbf{p}_1))_{\mathbf{B}^3\mathbf{U}(1)}$ $\simeq \text{Map}(X, \mathbf{BString}^{\mathbf{c}_2}(n))$	twisted String 2-bundles (heterotic Green-Schwarz mech.)



# Higher symmetry – Example: GS-mechanism. [SSS09][FSS12][FSS20-M5Str][SS20-M5GS]

$$\text{Under } \begin{array}{c} \text{Lie groups} \\ \text{Grp}(\text{SmthMfd}) \end{array} \begin{array}{c} \longleftarrow \\ \longrightarrow \end{array} \begin{array}{c} \text{diffeological groups} \\ \text{Grp}(\text{DfflSpc}) \end{array} \begin{array}{c} \longleftarrow \\ \longrightarrow \end{array} \begin{array}{c} \text{smooth } \infty\text{-groups} \\ \text{Grp}(\text{SmthGrpd}_\infty) \end{array}$$

mapping stack into delooping...

is moduli stack of:

$$\text{Map}(X, \mathbf{BU}(1))$$

circle bundles

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bundle  $p$ -gerbes

$$\text{Map}(X, \mathbf{B}\text{Spin}(n))$$

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String 2-bundles

$$\simeq \text{Map}(X, \mathbf{B}\text{String}(n))$$

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twisted String 2-bundles  
(heterotic Green-Schwarz mech.)

$$\simeq \text{Map}(X, \mathbf{B}\text{String}^{\mathbf{c}_2}(n))$$

**Rem.:** Different smooth  $\infty$ -groups  $\mathcal{G}$  may have same shape  $\int \mathcal{G}$  discrete  $\infty$ -group, e.g:

$$\mathbf{BPU}_\omega \xrightarrow{\eta^f} \mathbf{B}^3\mathbb{Z} \xleftarrow{\eta^f} \mathbf{B}^2\mathbf{U}(1)$$

$$\quad \quad \quad \wr \downarrow$$

$$\quad \quad \quad K(\mathbb{Z}, 3)$$

## Higher symmetry – Example: Cohomotopy.

Exs. 2.10, 2.16, 4.16 in [FSS20-Char]

Recall that *every connected* space is the classifying space of its loop  $\infty$ -group.

E.g., the 4-sphere encodes a rich  $\infty$ -higher symmetry

$$S^4 \simeq B(\Omega S^4) \in \mathbf{Grpd}_\infty, \quad \Omega S^4 \in \mathbf{Grp}(\mathbf{Grpd}_\infty).$$

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The corresponding moduli are classified by unstable/non-abelian **Cohomotopy**:

$$\pi^4(\mathbf{X}) := \pi_0 \mathbf{Map}(\mathbf{X}, S^4) \simeq \pi_0 \mathbf{Map}(\mathbf{X}, B(\Omega S^4)) \simeq H^1(\mathbf{X}; \Omega S^4)$$

4-Cohomotopy

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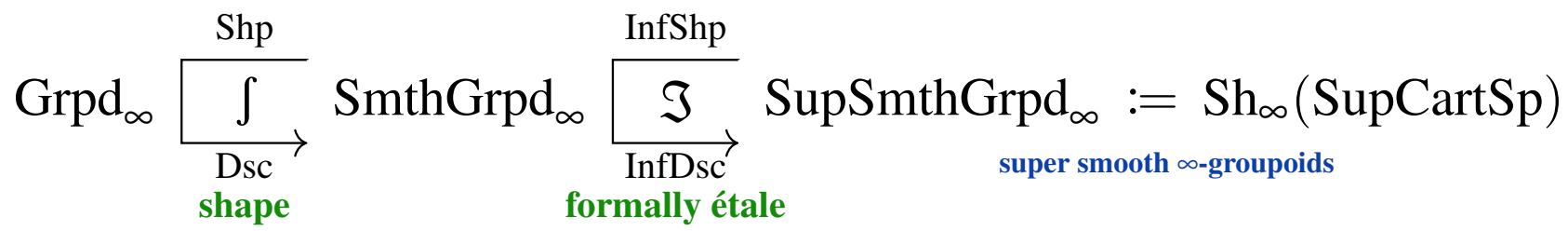
Incidentally, on 10-manifolds  $X^{10}$ , 4-Cohomotopy is stably equivalent to  $\text{tmf}^4$  (cf below):

$$\begin{array}{ccccc}
 S^4 & \xrightarrow{\quad\quad\quad} & \Omega^\infty \Sigma^\infty S^4 = \Omega^\infty S^4 & \xrightarrow[\sim_{\leq 10}]{\beta} & \Omega^\infty \text{tmf}^4 \\
 & & \text{stabilization/} & & \text{Boardman homomorphism} \\
 & & \text{abelianization} & & \\
 \pi^4(\mathbb{R}^{0,1} \times X^{10}) & \xrightarrow{\quad\quad\quad} & S^4(\mathbb{R}^{0,1} \times X^{10}) & \xrightarrow{\sim} & \text{tmf}^4(\mathbb{R}^{0,1} \times X^{10}) \\
 \text{unstable/} & & \text{stable/} & & \text{elliptic 4-cohomology} \\
 \text{non-abelian} & \text{4-Cohomotopy} & \text{abelianized} & \text{4-Cohomotopy} & 
 \end{array}$$

Higher geometry locally modeled on

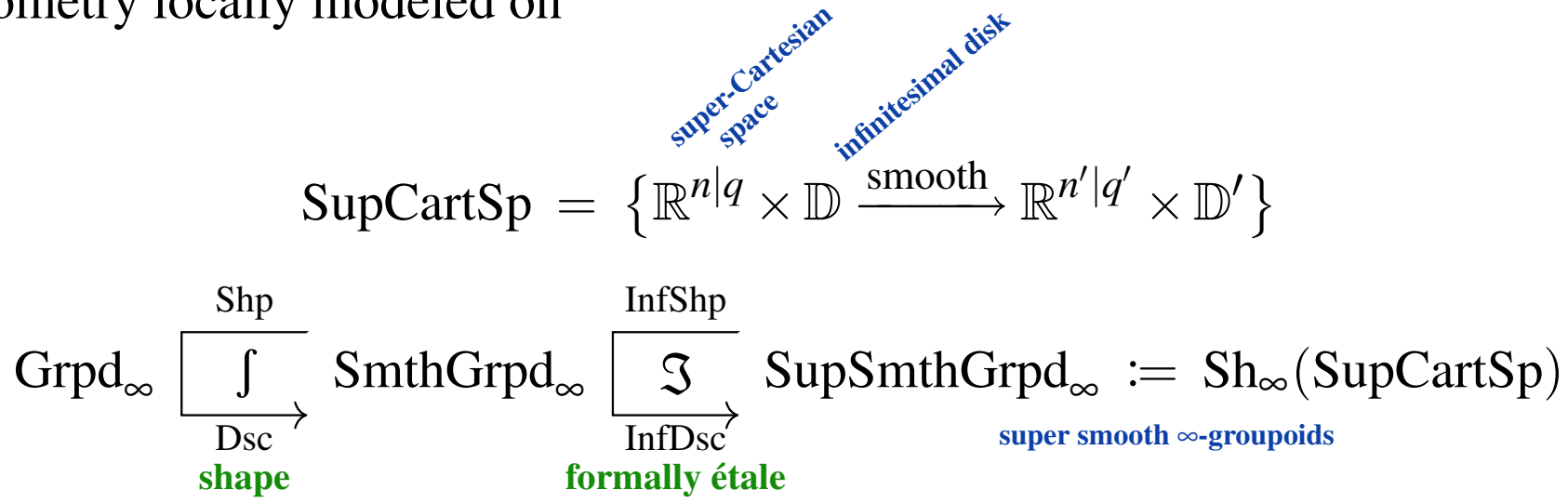
$$\text{SupCartSp} = \left\{ \mathbb{R}^{n|q} \times \mathbb{D} \xrightarrow{\text{smooth}} \mathbb{R}^{n'|q'} \times \mathbb{D}' \right\}$$

super-Cartesian space
infinitesimal disk



lifts all fundamentals of differential geometry to higher geometry of super  $\infty$ -stacks, e.g.:

Higher geometry locally modeled on



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<p>A morphism of super <math>\infty</math>-stacks is a <b>local diffeomorphism</b> or <i>formally étale</i> if its <math>\mathfrak{S}</math>-unit is homotopy cartesian:</p>	$\begin{array}{ccc} X & \xrightarrow{\eta^{\mathfrak{S}}} & \mathfrak{S}X \\ f \downarrow \text{ét} & \text{(pb)} & \downarrow \mathfrak{S}f \\ Y & \xrightarrow{\eta^{\mathfrak{S}}} & \mathfrak{S}Y \end{array}$
<p>The <b>infinitesimal neighbourhood</b> around point <math>x</math> in a super <math>\infty</math>-stack <math>X</math> is the <math>x</math>-fiber of the <math>\mathfrak{S}</math>-unit:</p>	$\begin{array}{ccc} \mathbb{D}_x X & \longrightarrow & * \\ \downarrow & \text{(pb)} & \downarrow \mathfrak{S}_x \\ X & \xrightarrow{\eta^{\mathfrak{S}}} & \mathfrak{S}X \end{array}$

For  $V \in \text{Grp}(\text{SupSmthGrpd}_\infty)$  a super group stack such as super-Minkowski  $V = \mathbb{R}^{d,1|\mathbf{N}}$ :

**Def.:** A  $V$ -fold is an étale  $\infty$ -stack locally diffeomorphic to  $V \xleftarrow[\text{ét}]{V\text{-atlas}} U \xrightarrow[\text{eff.epi}]{V\text{-fold}} X$ .

(This cohesive **higher Cartan geometry** is formalized in Modal Homotopy Type Theory:  
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$$\text{Map}(X, \mathbf{B}\mathcal{G})_{\mathbf{BAut}(\mathbb{D}_e V)} = \left\{ \begin{array}{ccc} X & \overset{\tau}{\dashrightarrow} & \mathbf{B}\mathcal{G} \\ \swarrow \vdash\text{Fr}(X) & \overset{\sim}{\rightleftarrows} \text{vielbein} & \searrow \mathbf{Bstr} \\ & \mathbf{BAut}(\mathbb{D}_e V) & \end{array} \right\}$$

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**So** for  $X$  a  $\mathbb{R}^{d,1|\mathbf{N}}$ -fold we have moduli of **super-vielbein fields**:

$$\text{Map}(X, \mathbf{BSpin}(d))_{\mathbf{BGL}(d,1|\mathbf{N})} = \left\{ \begin{array}{ccc} X & \overset{\tau}{\dashrightarrow} & \mathbf{BSpin}(d) \\ \swarrow \vdash \text{Fr}(X) & \begin{array}{c} \overset{\sim}{\rightleftarrows} \\ \text{vielbein} \end{array} & \searrow \\ & \mathbf{BGL}(d|\mathbf{N}) & \end{array} \right\}$$

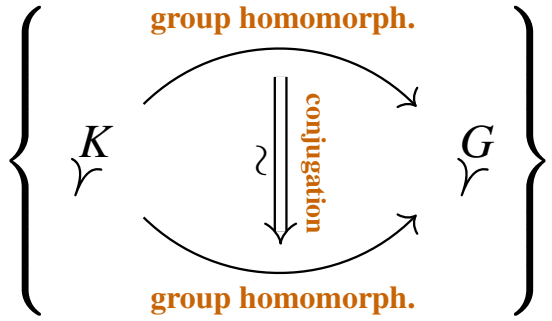
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# Singular-Cohesive Homotopy Theory of orbi- $\infty$ -stacks.

[Rezk14][SS20-OrbCoh]

Higher geometry locally modeled on orbi-singularities:

$$\text{Snglrt} := \left\{ \underset{\mathcal{Y}}{G} \mid G \text{ fin. group} \right\} \quad \text{with} \quad \text{Map}\left(\underset{\mathcal{Y}}{K}, \underset{\mathcal{Y}}{G}\right) =$$



$$\mathbf{H} = \text{GloSupSmthGrpd}_\infty := \text{Sh}_\infty(\text{SupCartSp} \times \text{Snglrt})$$

orbi-singular super- $\infty$ -stacks

faithfully subsumes proper equivariant homotopy theory:

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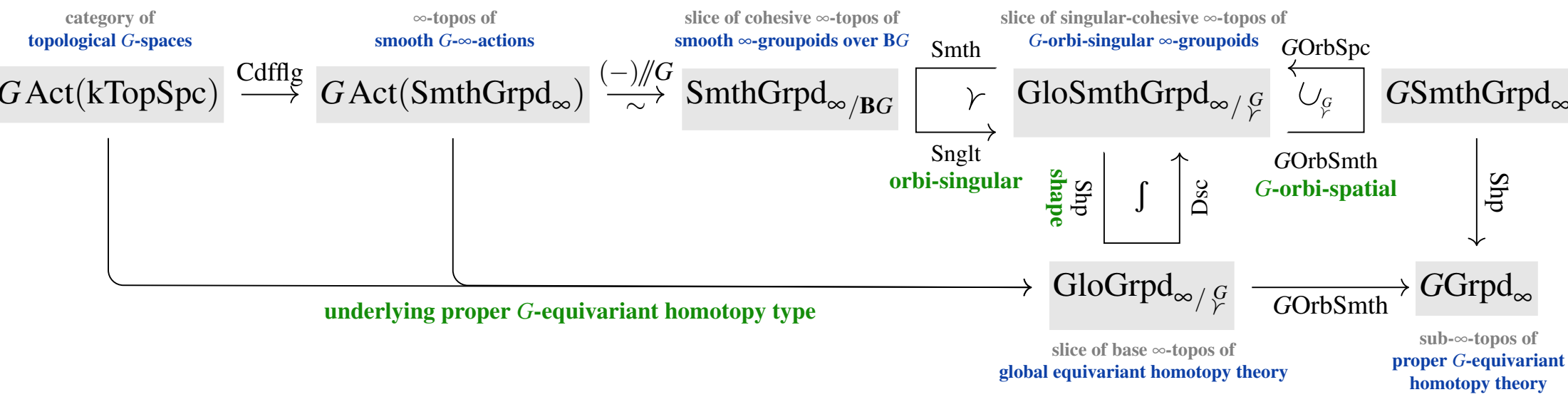
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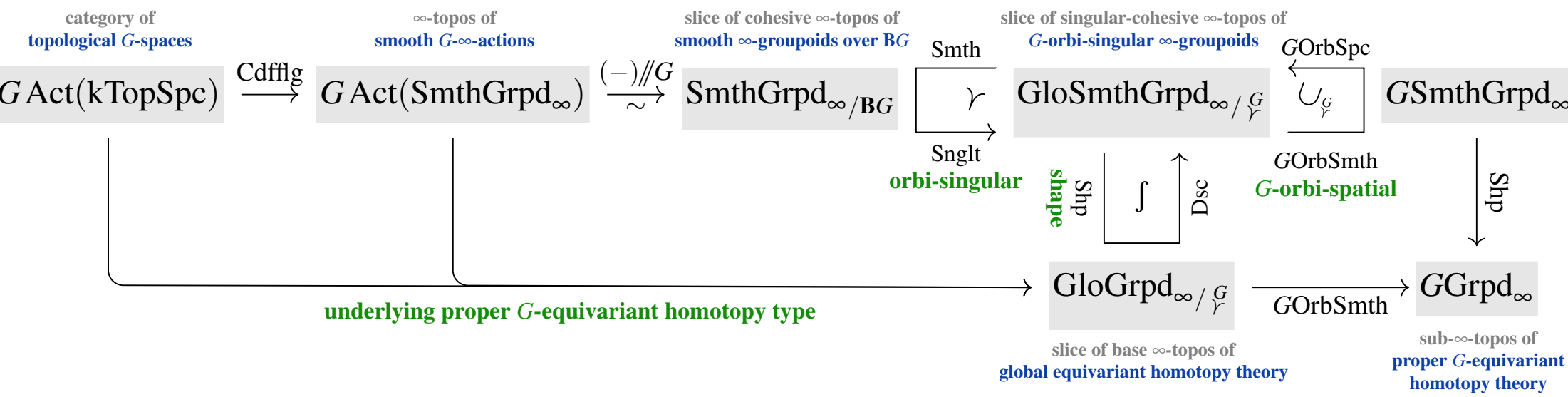
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**Thm.** (§4.1 in [SS20-OrbCoh])

Good orbifolds covered by  $G \curvearrowright X$  are equivalently  $\gamma(X//G) \in \mathbf{H} = \text{GloSmthGrpd}_\infty$  and their proper-equivariant homotopy type is:

$$\int \gamma(X//G) \simeq \text{GOrbSpc} \left( \int (X^{(-)}) \right) \in \text{GGrpd}_\infty \xrightarrow{\text{DscGOrbSpc}} \mathbf{H}/\gamma$$

**Theorem.**

If  $G \curvearrowright \Gamma$  is a  $G$ -equivariant Hausdorff-topological group with  $\int \Gamma$  truncated, then

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 B_G \Gamma
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classifies  $G$ -equivariant  $\Gamma$ -principal bundles on  $G$ -orbifolds  $\mathcal{X} \simeq \gamma(X // G) \in \mathbf{H}_{/\gamma}^G$ :

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and its equivariant homotopy groups are given by non-abelian group cohomology:

$$\pi_{\bullet}^H(B_G \Gamma) \simeq H_{\text{Grp}}^{1-\bullet}(H; \int \Gamma)$$

*equivariant homotopy groups*      *equivariant classifying shape*      *non-abelian group cohomology*      *isotropy group*      *shape of structure group*



Specifically, for

$$1 \rightarrow N \hookrightarrow G \twoheadrightarrow \mathbb{Z}_2 \rightarrow 1$$

and

$$\mathbb{Z}_2 \curvearrowright \mathrm{PU}_\omega^{\mathrm{gr}} \in G\mathrm{Act}(\mathrm{Grp}(\mathrm{kTopSpc}))$$

the graded projective unitary group acted on by complex conjugation, the  $G$ -orbi-space

$$B_G(\mathrm{PU}_\omega^{\mathrm{gr}}) \in \mathbf{H}$$

classifies type **IIA**  $B$ -fields on  $G$ -orbi-orientifolds

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**Philosophical question:**

But *why* coefficients like  $B_G \mathrm{PU}_\omega$ ? *Are there god-given coefficients?*



For any line object  $\mathbb{A}^1$  there are the *Tate spheres* (e.g. [VRO07, Rem. 2.22])

$$S_{\text{Tate}}^n := \text{cof}(\mathbb{A}^n \setminus \{0\} \hookrightarrow \mathbb{A}^n) \in \mathbf{H}$$

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Their shape is that of the ordinary  $n$ -spheres ([SS20-OrbCoh, Ex. 5.21]):

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More generally, for any

$$G \curvearrowright V \in G\text{Act}(\text{VectorSpaces}_\mathbb{R}) \hookrightarrow G\text{Act}(\text{SmthMfd}) \hookrightarrow G\text{Act}(\mathbf{H})$$

we have the **orbi-smooth  $V$ -Tate spheres** ([SS20-OrbCoh, Ex. 5.27])

$$\int \gamma(S_{\text{Tate}}^V // G) \in \mathbf{H}_{/\gamma}^G.$$

---

The  $G_{\text{ADE}}$ -equivariant Tate 4-sphere has equivariant homotopy type of the 4-representation sphere:

$$\int \mathcal{Y}(S_{\text{Tate}}^4 // G_{\text{ADE}}) \underset{\substack{\text{stereogr.} \\ \text{project.}}}{\cong} S(\mathbb{R} \oplus \overset{G_{\text{ADE}}}{\mathbb{H}}) \in G_{\text{ADE}} \text{Grpd}_{\infty} \xrightarrow{\text{Dsc } G_{\text{ADE}} \text{ OrbSpc}} \mathbf{H} / \mathcal{Y}_{G_{\text{ADE}}}$$

**Example: The ADE-equivariant 4-sphere.**

§5.1 in [HSS18-ADE]; §3 in [SS19-TadCnc]

Consider the left multiplication action of  $\mathrm{Sp}(1) = S(\mathbb{H})$  on the quaternions  $\mathbb{H}$ :

$$\mathrm{Sp}(1) \curvearrowright \mathbb{H} \simeq \mathrm{SU}(2)_L \curvearrowright \mathbb{C}^2 \simeq \mathrm{Spin}(3)_L \curvearrowright \mathbb{R}^4.$$

The finite subgroups have a famous ADE-classification:

Label	$G_{\mathrm{ADE}} \subset_{\mathrm{fin}} \mathrm{SU}(2)$	Order	Name
$\mathbb{A}_n$	$\mathbb{Z}_{n+1}$	$n$	Cyclic
$\mathbb{D}_{n+4}$	$2\mathbb{D}_{n+2}$	$4(n+2)$	Binary dihedral
$\mathbb{E}_6$	$2\mathbb{T}$	24	Binary tetrahedral
$\mathbb{E}_7$	$2\mathbb{O}$	48	Binary octahedral
$\mathbb{E}_8$	$2\mathbb{I}$	120	Binary icosahedral

Denote the restricted representation by  $\mathbf{4} := G_{\mathrm{ADE}} \curvearrowright \mathbb{R}^4 \in \mathrm{RO}(G_{\mathrm{ADE}})$ .

The  $G_{\mathrm{ADE}}$ -equivariant Tate  $\mathbf{4}$ -sphere has equivariant homotopy type of the  $\mathbf{4}$ -representation sphere:

$$\int \gamma(S_{\mathrm{Tate}}^{\mathbf{4}} // G_{\mathrm{ADE}}) \underset{\substack{\text{stereogr.} \\ \text{project.}}}{\simeq} S(\mathbb{R} \oplus \overset{G_{\mathrm{ADE}}}{\mathbb{H}}) \in G_{\mathrm{ADE}} \mathrm{Grpd}_{\infty} \xrightarrow{\mathrm{Dsc } G_{\mathrm{ADE}} \mathrm{OrbSpc}} \mathbf{H} / G_{\mathrm{ADE}}$$

**Example: Super-Minkowski orbifolds.**

Thm. 4.3 in [HSS18-ADE]

**Theorem.** Classification of subgroup actions of  $\text{Pin}^+(10, 1) \curvearrowright \mathbb{R}^{10,1|32}$  which fix  $\geq 1/4$ th of **32** such that all non-trivial subgroups have the same bosonic fixed locus:

Black brane species	BPS	Fixed locus in $\mathbb{R}^{10,1 32}$	Type of singularity in $\mathbb{R}^{10,1}$	Intersection law
				$\simeq \mathbb{R}^{1,1} \oplus \mathbb{R}^4 \oplus \mathbb{R}^4 \oplus \mathbb{R}^1$

**Elementary brane species**

**Simple singularities**

MO9	1/2	$\mathbb{R}^{9,1 16}$	$\mathbb{Z}_2$	$=$	—				$(\mathbb{Z}_2)_{\text{HW}}$
MO5	1/2	$\mathbb{R}^{5,1 2 \cdot 8}$	$\mathbb{Z}_2$	$\overset{\Delta}{\subset}$	—				$(\mathbb{Z}_2)_R \times (\mathbb{Z}_2)_{\text{HW}}$
MO1	1/2	$\mathbb{R}^{1,1 16 \cdot 1}$	$\mathbb{Z}_2$	$\overset{\Delta}{\subset}$	—	$(\mathbb{Z}_2)_L$	$\times$	$(\mathbb{Z}_2)_R$	$\times (\mathbb{Z}_2)_{\text{HW}}$
MK6	1/2	$\mathbb{R}^{6,1 16}$	$\mathbb{Z}_{n+1}, 2\mathbb{D}_{n+2}, 2T, 2O, 2I$	$\subset$	—			$\text{SU}(2)_R$	—
M2	1/2 = 8/16	$\mathbb{R}^{2,1 8 \cdot 2}$	$\mathbb{Z}_2$	$\overset{\Delta}{\subset}$	—	$\text{SU}(2)_L$	$\times$	$\text{SU}(2)_R$	—
M2	6/16	$\mathbb{R}^{2,1 6 \cdot 2}$	$\mathbb{Z}_{n+3}$	$\overset{\Delta}{\subset}$	—	$\text{SU}(2)_L$	$\times$	$\text{SU}(2)_R$	—
M2	5/16	$\mathbb{R}^{2,1 5 \cdot 2}$	$2\mathbb{D}_{n+2}, 2T, 2O, 2I$	$\overset{\Delta}{\subset}$	—	$\text{SU}(2)_L$	$\times$	$\text{SU}(2)_R$	—
M2	1/4 = 4/16	$\mathbb{R}^{2,1 4 \cdot 2}$	$2\mathbb{D}_{n+2}, 2O, 2I$	$\overset{(\text{id}, \tau)}{\subset}$	—	$\text{SU}(2)_L$	$\times$	$\text{SU}(2)_R$	—

$$\mathbb{R}^{10,1} \simeq_{\mathbb{R}} \mathbb{R}^{1,1} \oplus \mathbb{R}^4 \oplus \mathbb{R}^4 \oplus \mathbb{R}$$

$\text{SU}(2)_L$        $\text{SU}(2)_R$        $(\mathbb{Z}_2)_{\text{HW}}$



**Example: Super-Minkowski orbifolds.**

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$$\mathbb{R}^{10,1} \simeq_{\mathbb{R}} \mathbb{R}^{1,1} \oplus \overset{\text{SU}(2)_L}{\curvearrowright} \mathbb{R}^4 \oplus \overset{\text{SU}(2)_R}{\curvearrowright} \mathbb{R}^4 \oplus \overset{(\mathbb{Z}_2)_{\text{HW}}}{\curvearrowright} \mathbb{R}$$

**yields super-Minkowski orbifolds, e.g.:**

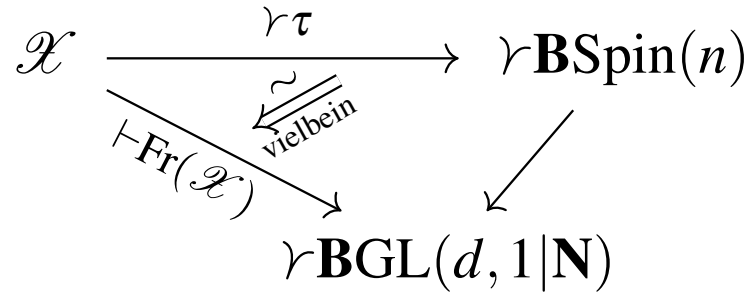
$$\mathcal{X}_{\text{MK6} \perp \text{K3}} = \mathbb{R}^{6,1|16} \times \gamma(\mathbb{T}^4 // \mathbb{Z}_2^A) \in \mathbf{H}_{\mathbb{Z}_2^A}$$

# J-twisted proper orbifold Cohomotopy.

Def. 5.28 in [SS20-OrbCoh]

Let  $\mathcal{X}$  be an  $\mathbb{R}^{d,1|\mathbf{N}}$ -orbifold

with  $\text{Spin}(n)$ -structure

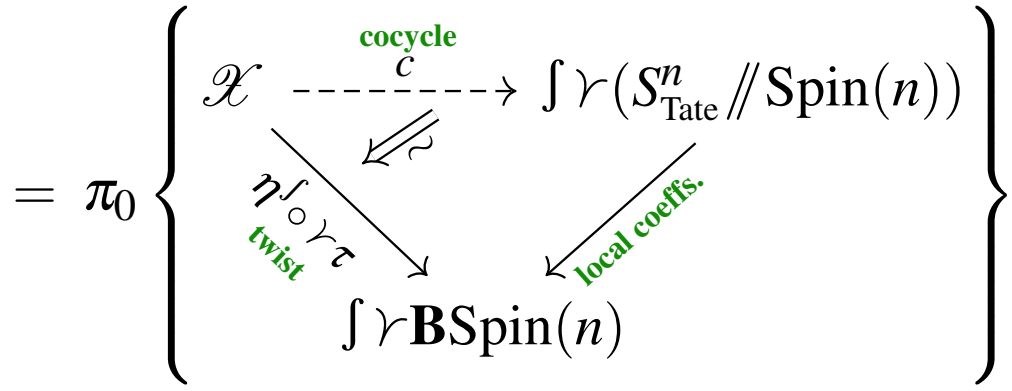


**Def.** ([SS20-OrbCoh, Ex. 5.29]) **J-twisted proper orbifold Cohomotopy** of  $(\mathcal{X}, \tau)$  is :

tangentially J-twisted proper orbifold Cohomotopy
proper equivariant homotopy type of canonical n-sphere

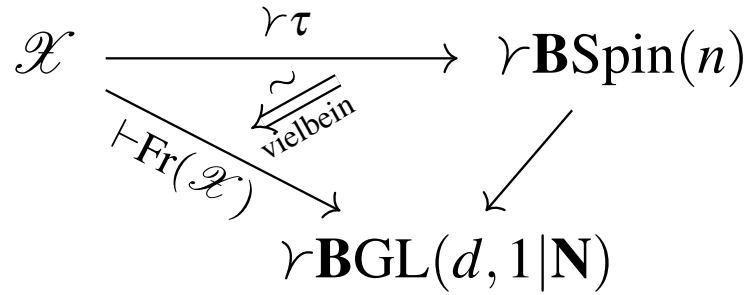
$$\pi^{\int \gamma\tau}(\mathcal{X}) := \pi_0 \text{Map} \left( \mathcal{X}, \int \gamma(S_{\text{Tate}}^n // \text{Spin}(n)) \right)_{\int \gamma\mathbf{BSpin}(d)}$$

proper equivariant tangential twist



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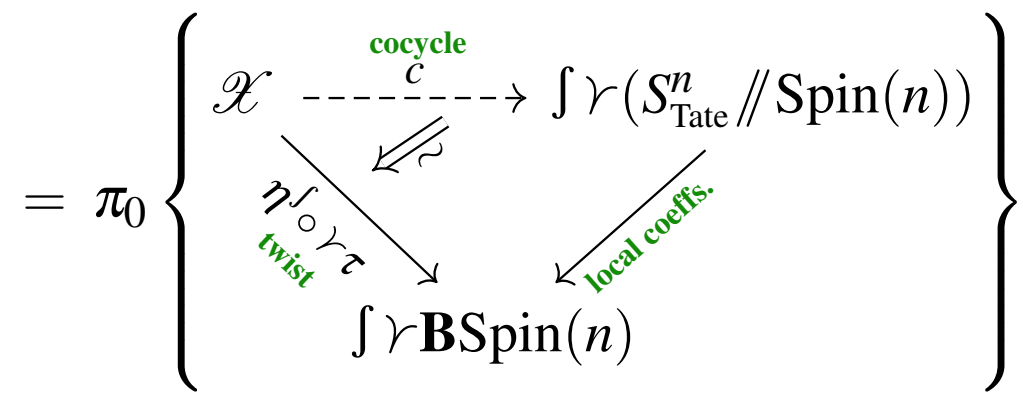
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proper equivariant tangential twist



In the case that  $\mathcal{X} = X$  is smooth (i.e. a manifold), this reduces to **J-twisted Cohomotopy**:  
 [FSS19-HypH, Def. 2.1][FSS20-Char, Ex. 2.41], see also [Cru03, Lem. 5.2].

$$\begin{array}{ccc}
 \mathcal{X} & \xrightarrow{\text{cocycle}} & \int \gamma (S_{\text{Tate}}^{\mathbf{n}|N} // \text{Spin}(n)) \\
 \searrow \eta \circ \gamma_\tau & \Downarrow \cong & \downarrow \\
 & & \int \gamma \mathbf{B}\text{Spin}(n)
 \end{array}$$

**Key Structure Theorem:**

In limiting cases this reduces to:

- (a) equiv Cohomotopy in RO-deg  $\mathbf{n}$
- (b) tangent J-twisted Cohomotopy

tangentially J-twisted orbifold Cohomotopy

$$\pi^{\int \gamma_\tau}(\mathcal{X})$$

(a) on flat orbifolds

$$\mathcal{X} = \gamma(\mathbb{T}^d // G)$$

(b) on smooth orbifolds

$$\mathcal{X} = X$$

- [HSS18-ADE]
- [SS19-TadCnc]
- [BSS19-FrcBrn]
- [SS20-M5GS]

$$\pi_G^{\mathbf{n}}(\mathbb{T}^d)$$

equivariant Cohomotopy in RO-degree  $\mathbf{n}$

- [FSS19-HypH]
- [FSS19-M5WZ]
- [FSS20-M5Str]
- [SS20-M5Anom]
- [FSS20-GSAnom]

$$\pi^\tau(X)$$

tangentially J-twisted Cohomotopy

$$\begin{array}{ccc}
 \gamma(\mathbb{T}^d // G) & \xrightarrow{\text{cocycle}} & \int \gamma (S^n // G) \\
 \searrow \gamma_\tau & \Downarrow \cong & \downarrow \\
 & & \gamma G
 \end{array}$$

$$\begin{array}{ccc}
 X & \xrightarrow{\text{cocycle}} & S^n // \text{Spin}(n) \\
 \searrow \int_\tau & \Downarrow \cong & \downarrow \\
 & & \mathbf{B}\text{Spin}(n)
 \end{array}$$

higher orbi-geometry

orbifold cohomology

I – Proper Orbifold Cohomology

II – **for M-Theory**

Hypothesis H

M5-brane charges in  
flat orbi-orientifolds

*Future historians may judge the late 20th century as a time when theorists were like children playing on the seashore, diverting themselves with the smoother pebbles or prettier shells of superstrings while the great ocean of M-theory lay undiscovered before them.*

M. Duff (1998)

closing sentence in:

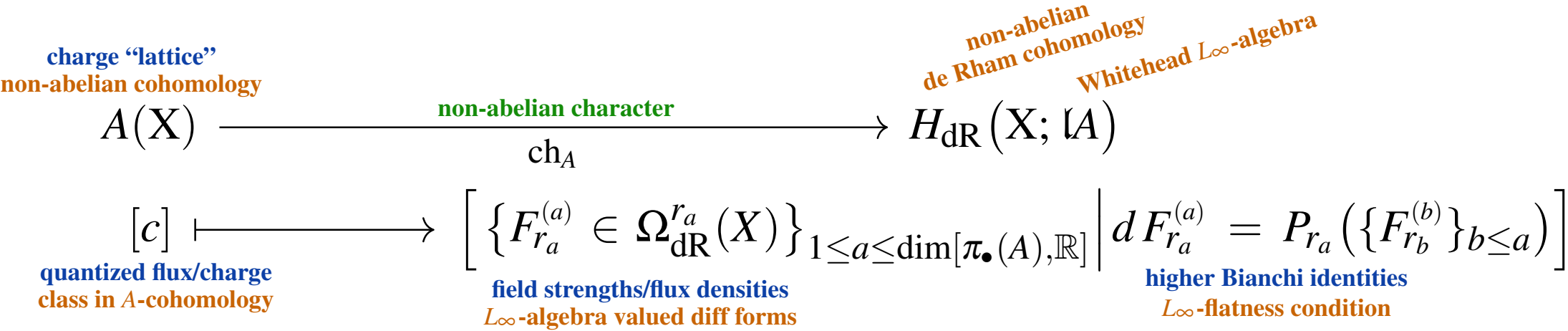
M. Duff:

*The Theory Formerly Known as Strings*

Scient Amer 1998

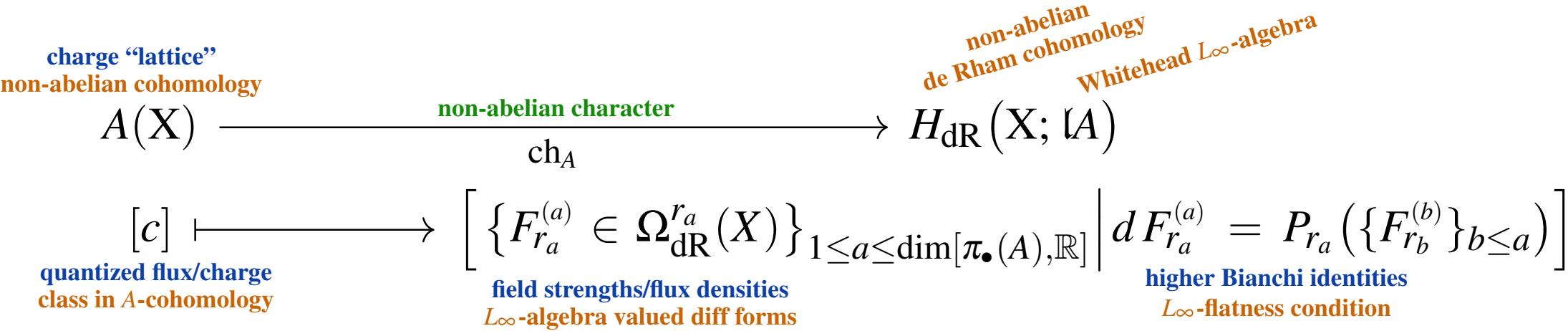
Non-perturbative completion of a theory of charged objects  
with flux densities satisfying Bianchi identities  
involves choosing a non-abelian cohomology theory  $A$   
whose character image enforces these Bianchi identities:

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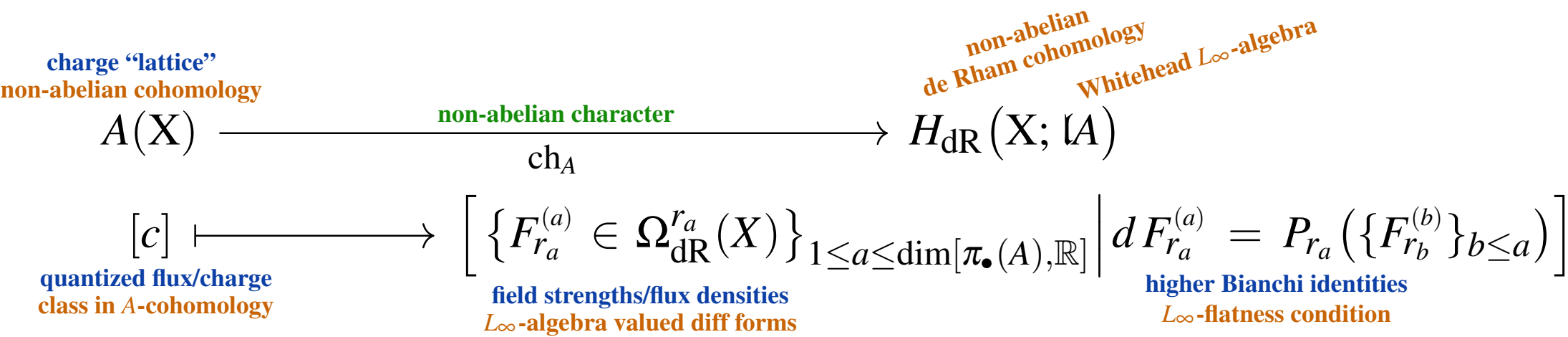


Non-perturbative completion of a theory of charged objects with flux densities satisfying Bianchi identities involves choosing a non-abelian cohomology theory  $A$  whose character image enforces these Bianchi identities:



The choice of  $A$  is a **hypothesis** about the correct non-perturbative completion.

Non-perturbative completion of a theory of charged objects with flux densities satisfying Bianchi identities involves choosing a non-abelian cohomology theory  $A$  whose character image enforces these Bianchi identities:



The choice of  $A$  is a **hypothesis** about the correct non-perturbative completion.

Given such a choice, the moduli  $\infty$ -stack of fields is a differential refinement of  $A$ :

$$\mathcal{A} = \widehat{A} \in \text{SmthGrpd}_\infty$$

**Fact:** The Bianchi identity of the **type IIA RR/B-fields**

is that enforced by the Whitehead  $L_\infty$ -algebra of twisted KU-theory

$$H_{\text{dR}}(X, \mathfrak{l}(\text{KU} // \text{BU}(1))) \simeq \left\{ \left( \begin{array}{c} \{F_{2k}\}_k \\ H_3 \end{array} \right) \in \Omega_{\text{dR}}^\bullet(X) \mid \begin{array}{l} dF_{2k} = H_3 \wedge F_{2k-2} \\ dH_3 = 0 \end{array} \right\} \sim_{\text{conc}}$$

The evident **hypothesis** here is the proposal by Minasian/Moore/Witten/Bouwknegt/Mathai:

*The type IIA RR/B-field is flux-quantized in twisted K-theory.*

It *must* be flux-quantized in something at least close, such as orbifold KR-theory.

---

## Hypothesis H.

[Sa13, §2.5][FSS16-RatCoh][FSS19-HypH][SS21-MF]

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**Fact:** The Bianchi identity of the **M-theory C-field** is

that enforced by the Whitehead  $L_\infty$ -algebra of 4-Cohomotopy:

$$H_{\text{dR}}(\mathbf{X}, \mathfrak{l}S^4) \simeq \left\{ \left( \begin{array}{c} G_7 \\ G_4 \end{array} \right) \in \Omega_{\text{dR}}^\bullet(\mathbf{X}) \left| \begin{array}{l} dG_7 = -\frac{1}{2}G_4 \wedge G_4 \\ dG_4 = 0 \end{array} \right. \right\} \sim_{\text{conc}}$$

The evident **hypothesis** here [Sa13, §2.5] we called Hypothesis H:

*The M-theory C-field is flux-quantized in 4-Cohomotopy.*

It *must* be charge-quantized in something at least close, such as J-twisted orbifold Cohomotopy.

# Approximating Cohomotopy by K-theory.

[BSS19-FrcBrn][SS19-TadCnc][SS21-MF]

J-twisted orbifold Cohomotopy  
around an orbi-singularity

$$\pi_G^4(\mathbb{R}_{\text{cpt}}^4)$$

un-stable/  
non-linear!

equivariant generalized  
cohomologies in RO-degree 4

equivariant generalized  
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representation  
rings

# Approximating Cohomotopy by K-theory.

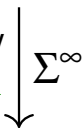
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$$\mathbb{S}_G^4(S^4) \quad \text{=====} \quad \mathbb{S}_G^0$$

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$$\xrightarrow[\text{[De06][Gui06]}]{\text{[BP72][Se74]}}$$

$$\mathbf{R}_{\mathbb{F}_1}(G)$$

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**Rem.** [FSS20-Char, (353)].

The Boardman homomorphism exhibits exactly the identification  $G_4 \mapsto F_4$  of [DMW00]:

$$\begin{array}{ccccccc}
 \text{ch} \left\{ \begin{array}{l} \xrightarrow{\pi^4} \\ \xrightarrow{(G_4, G_7)} \end{array} \right. & \xrightarrow[\text{stabilization / linearization}]{\Sigma^\infty} & S^4 & \xrightarrow[\text{Boardman homomorphism}]{\beta} & \text{KU}^4 & \xrightarrow[\text{Bott per.}]{=} & \text{KU} \\
 & & \xrightarrow{\text{C-field flux}} & & \xrightarrow{\text{RR-field flux}} & & \\
 & & G_4 & \xrightarrow{\text{}} & F_4 & & 
 \end{array}$$

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Rem. [FSS20-Char, (353)].

The Boardman homomorphism exhibits exactly the identification  $G_4 \mapsto F_4$  of [DMW00]:

$$\begin{array}{ccccccc} \text{ch} \left\{ \begin{array}{l} \xrightarrow{\pi^4} \\ \xrightarrow{(G_4, G_7)} \end{array} \right. & \xrightarrow[\text{stabilization / linearization}]{\Sigma^\infty} & \mathbb{S}^4 & \xrightarrow[\text{Boardman homomorphism}]{\beta} & \text{KU}^4 & \xrightarrow[\text{Bott per.}]{=} & \text{KU} \\ & \mapsto & G_4 & \mapsto & F_4 & & \\ & & \text{C-field flux} & & \text{RR-field flux} & & \end{array}$$

However,  $\beta$  (and [DMW00]) misses the double dimensional reductions  $G_4 \mapsto H_3$  and  $G_7 \mapsto F_6$ ; these do appear from Cohomotopy via *cyclification* ([FSS16-RatCoh][FSS16-TDual][BSS19-RatSt]).

# Example: M5-tadpole cancellation.

[SS19-TadCnc]

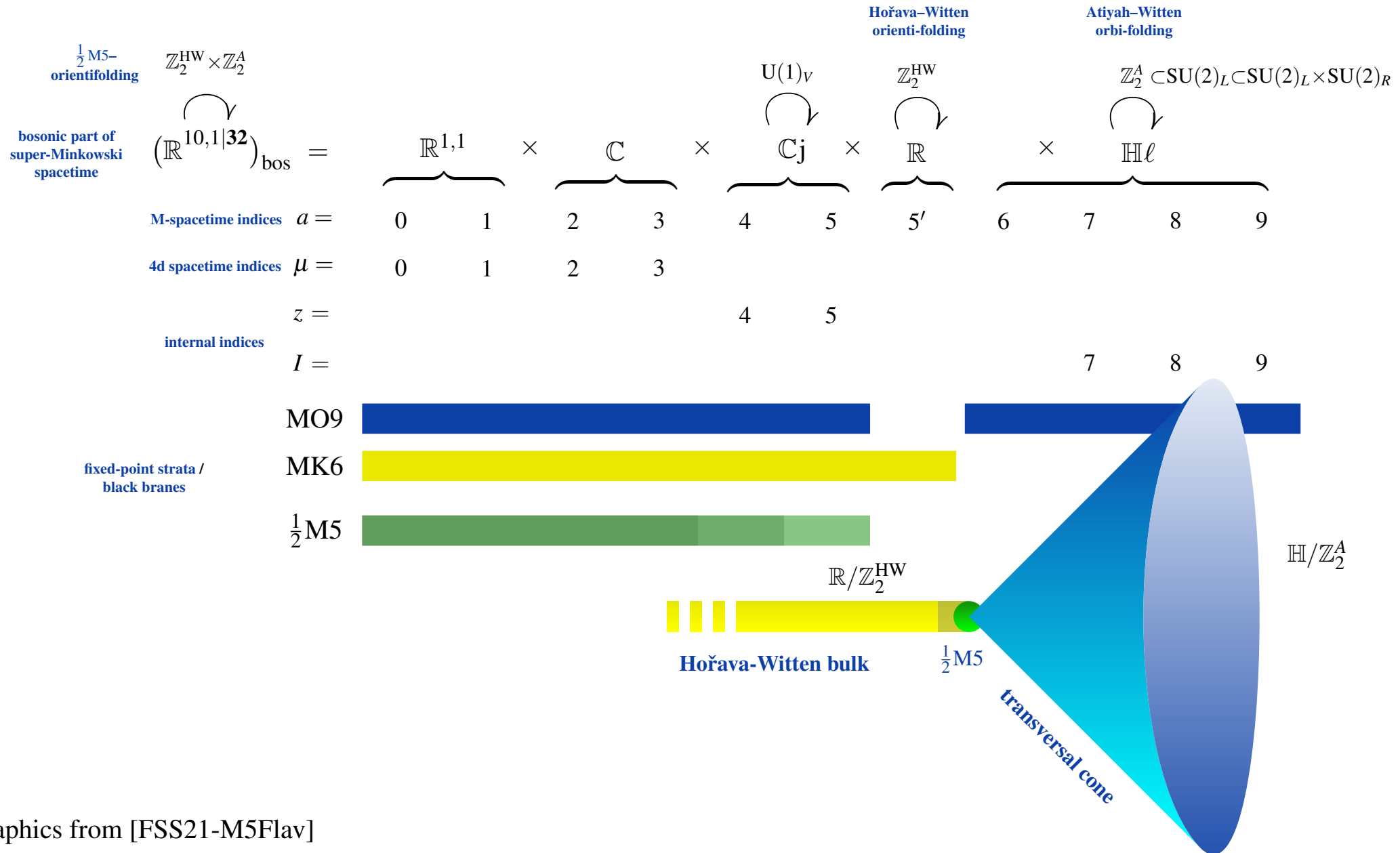
$$\pi^{\int \gamma(\text{Fr})} \left( \mathbb{R}^{5,1|8} \times \gamma(\mathbb{T}^4 // \mathbb{Z}_2^A) \right) \simeq \pi_{\mathbb{Z}_2^A}^4(\mathbb{T}^4) \simeq \left\{ N_{M5} \cdot \mathbf{2} - N_{MO5} \cdot \mathbf{1} \mid \begin{array}{l} N_{MO5} \in \{0, \dots, 16\} \\ N_{M5} \in \mathbb{Z} \end{array} \right\} \leftarrow \text{non-additive!}$$

heterotic M5-charges according to Hypothesis H

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[SS19-TadCnc, Cor. 4.6]



## Example: M5-tadpole cancellation.

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heterotic M5-charges according to Hypothesis H      equivariant 4-Cohomotopy      coincides with informal folklore [SS19-TadCnc, Cor. 4.6]

---

We now explain this example in more detail.

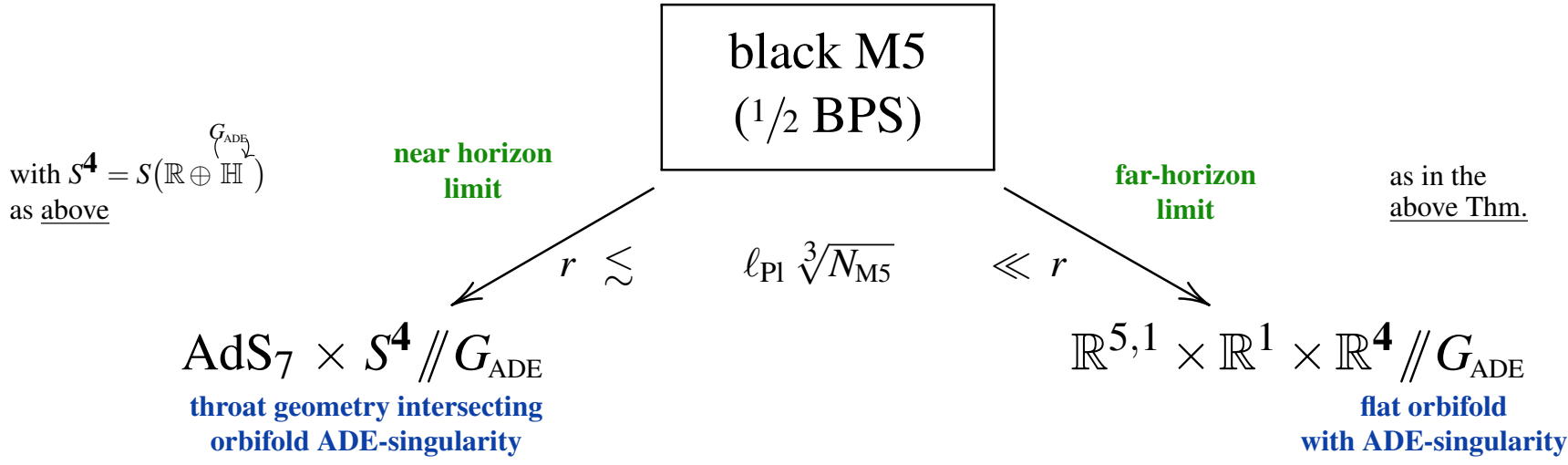


# Background: The black M5 in 11d SuGra.

---

**Fact.** [AFFH99, 5.2] [dMFF12, §8.3]:

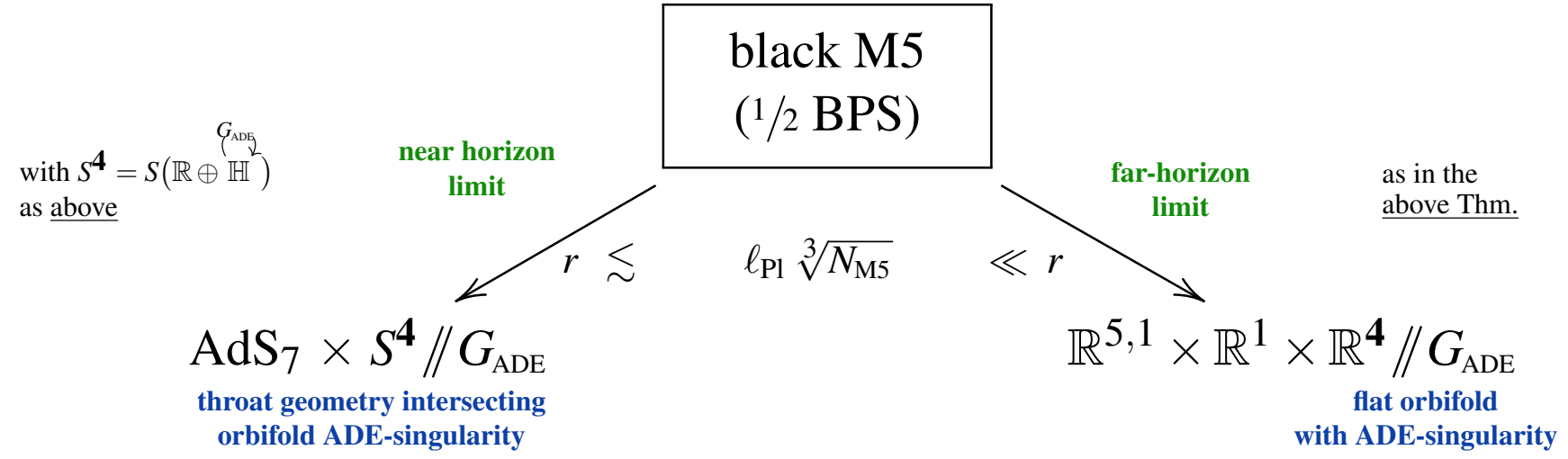
All BPS black M5-brane solutions of 11D supergravity are  $1/2$  BPS of this form:



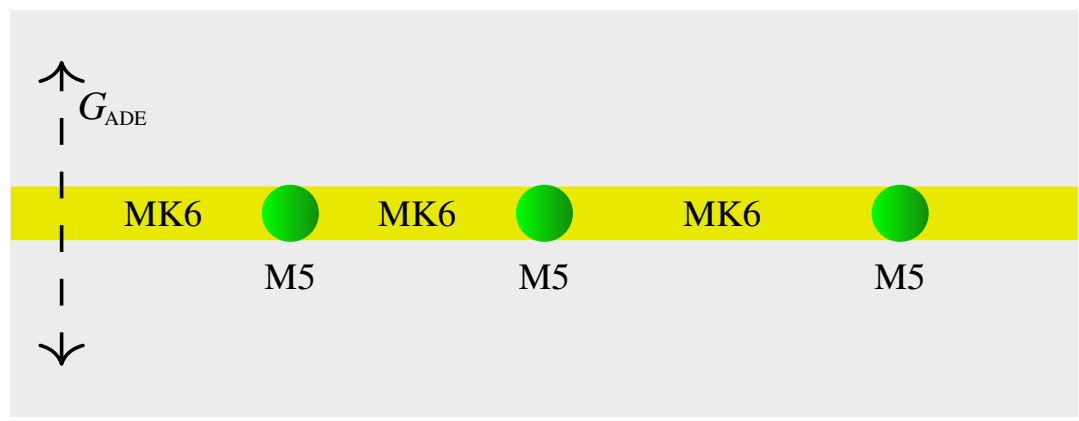
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**Fact.** [AFFH99, 5.2] [dMFF12, §8.3]:

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**Consequence 1:** Black BPS M5-branes are always domain walls inside an MK6-singularity:



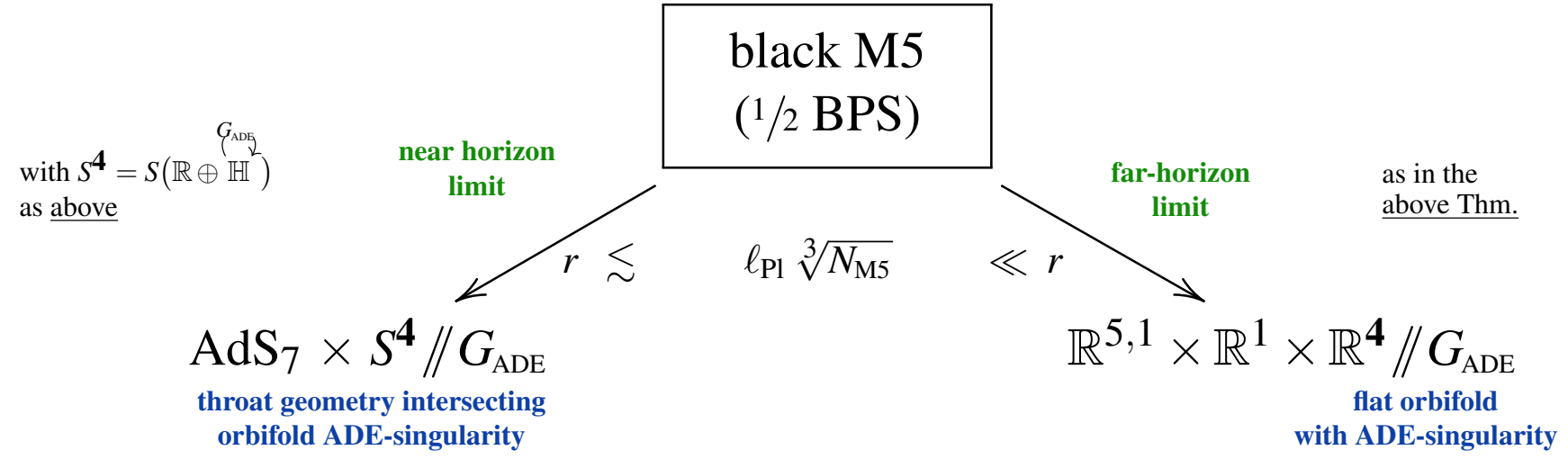
E.g.: [ZHTV14, §3.1] [Fa17, §3.3.1]



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All BPS black M5-brane solutions of 11D supergravity are  $1/2$  BPS of this form:



## Consequence 2:

Individual M5-branes  $N_{\text{M5}} \sim \mathcal{O}(1)^3$  have Planck scale thickness  $r \sim \ell_{\text{Pl}}$  hence their **near geometry make no sense** as solutions of M-theory due to infinite + unknown tower of higher curvature quantum corrections  $\sim (\ell_{\text{Pl}}^2 \cdot R)^k$ .

Planck area  
 Riem. curvature

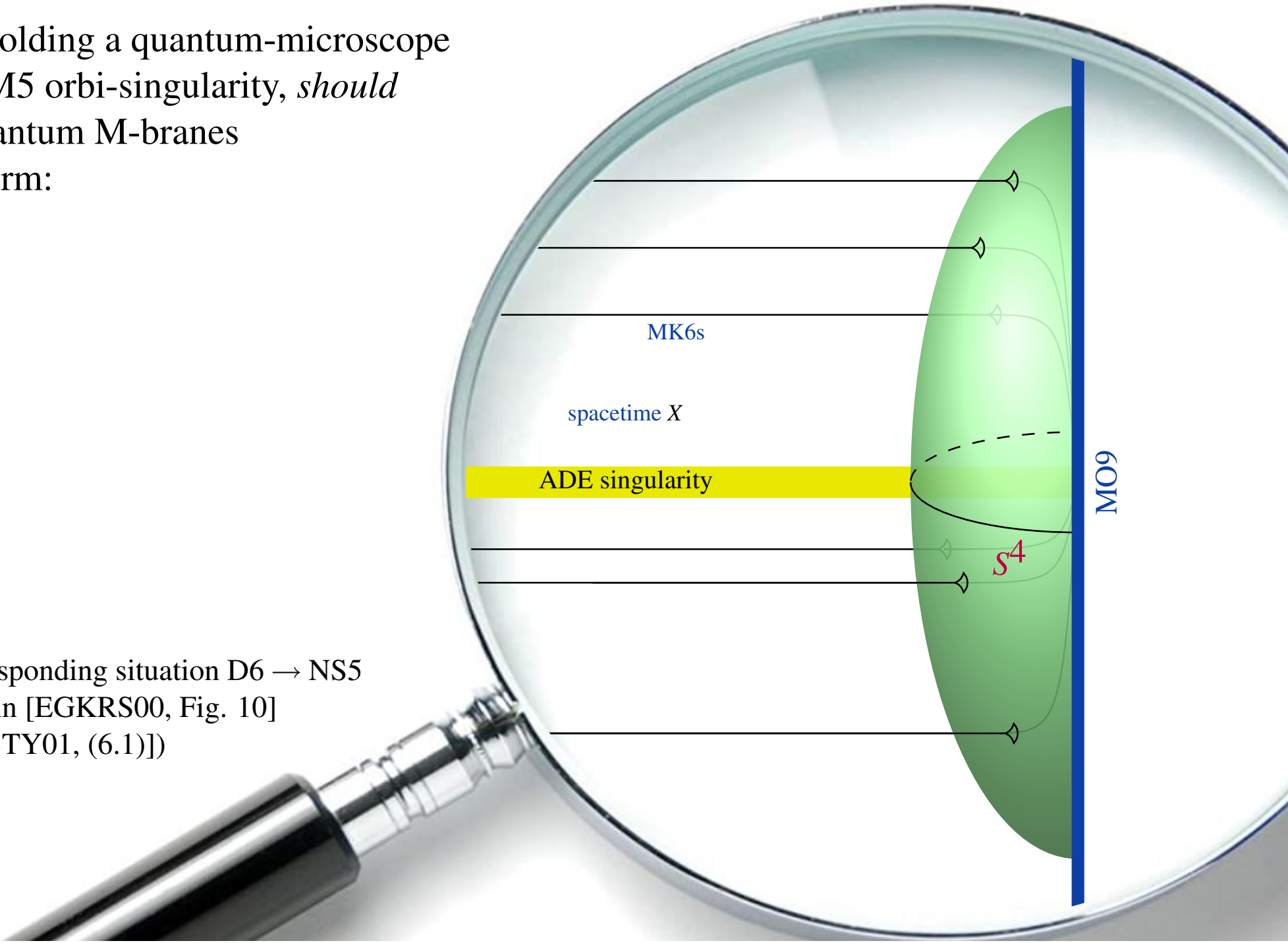
Conversely:

The **M-meaningful far geometry** yields flat super-orbifold spacetimes where all curvature is crammed into orbi-singularities so that also all quantum effects must be hiding inside orbi-singularities – plausibly detected as charges measured in a proper orbifold cohomology theory!



# Quantum M-branes?

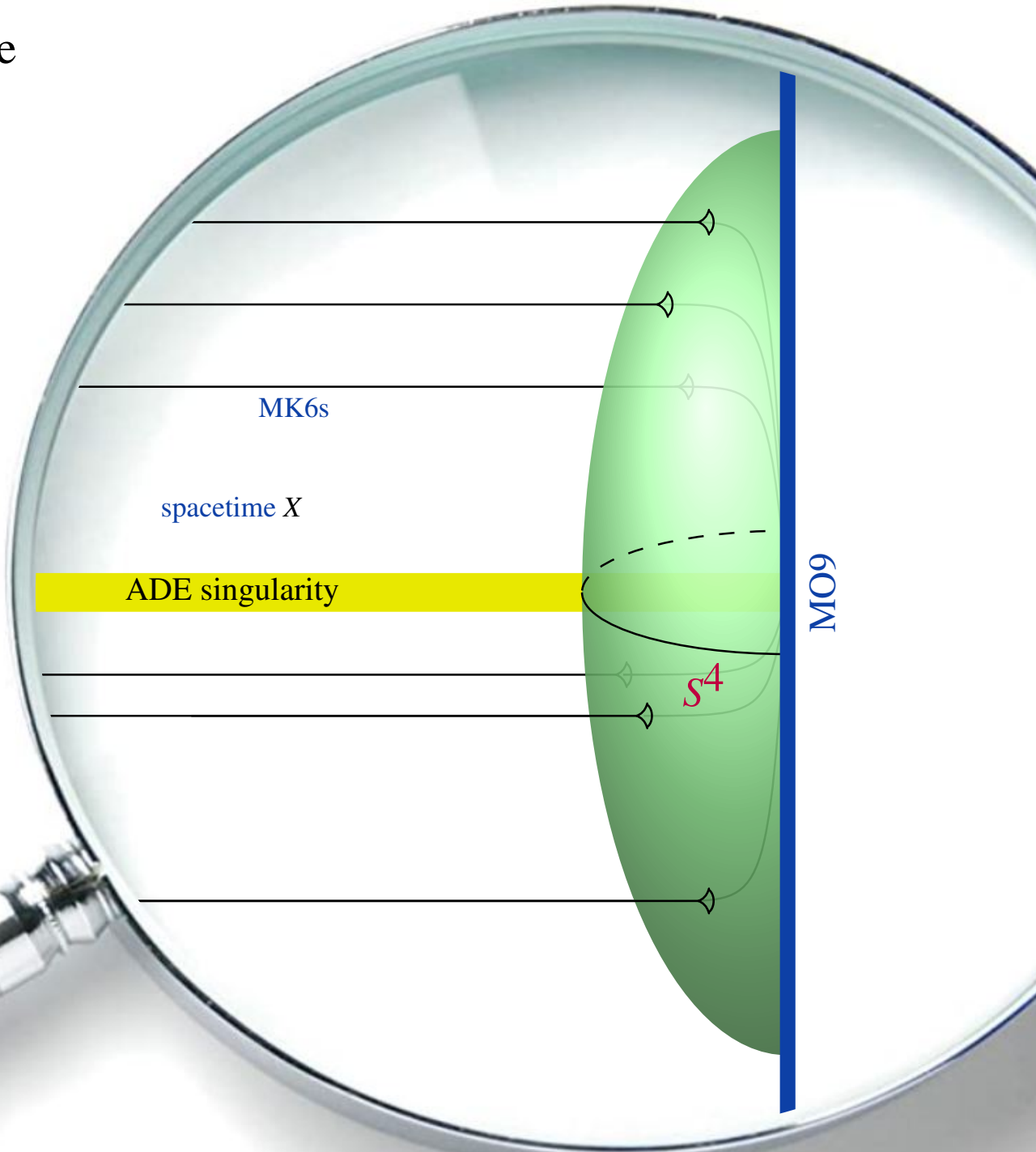
Hence, holding a quantum-microscope over a  $\frac{1}{2}$ M5 orbi-singularity, *should* show quantum M-branes of this form:



(the corresponding situation  $D6 \rightarrow NS5$  is shown in [EGKRS00, Fig. 10] and [GKSTY01, (6.1)])

Hence, holding a quantum-microscope over a  $\frac{1}{2}$ M5 orbi-singularity, *does* show quantum M-branes of this form:

Hypothesis H asserts that:  
*This quantum-microscope is J-twisted Cohomotopy theory.*



(the corresponding situation  $D6 \rightarrow NS5$  is shown in [EGKRS00, Fig. 10] and [GKSTY01, (6.1)])

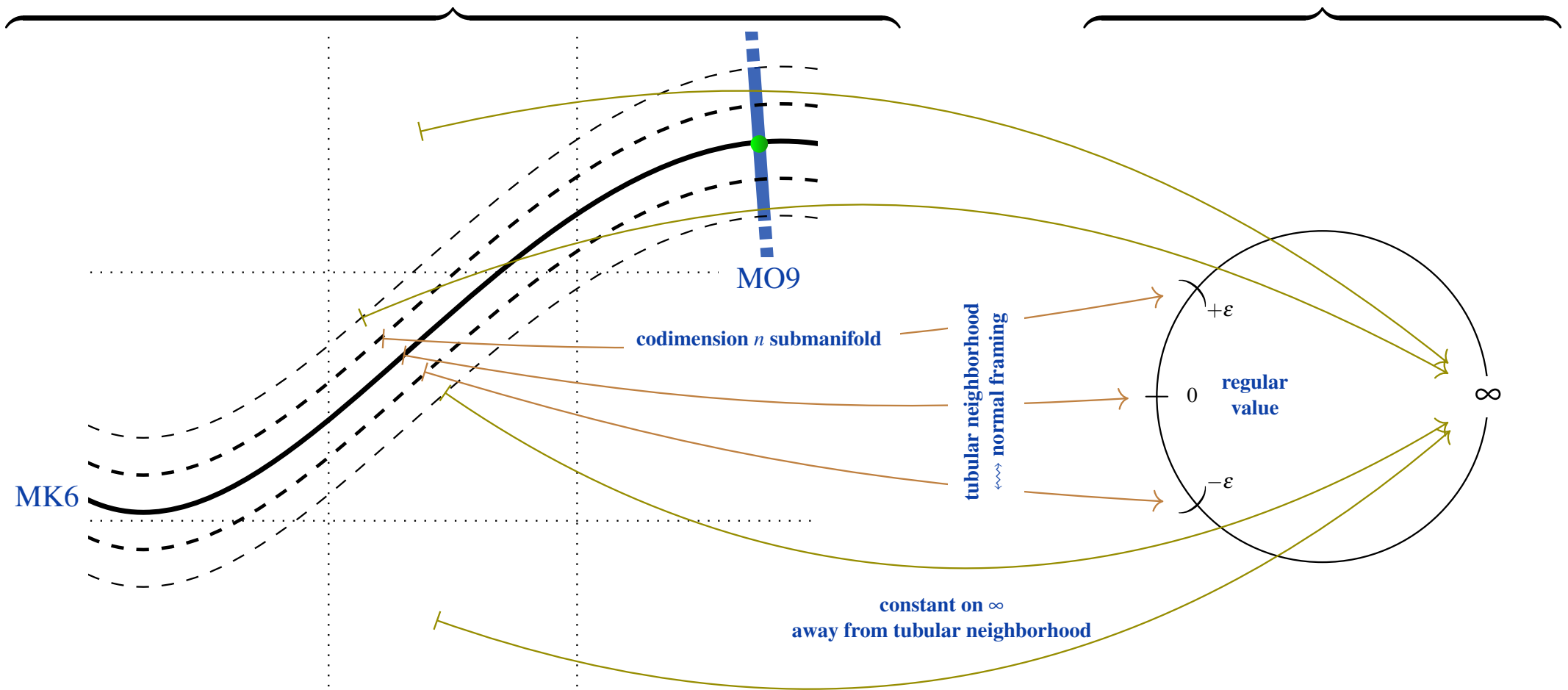
# Cohomotopy charge map.

[SS19-TadCnc, §2.1] [SS21-MF, §2.2]

Namely, a small tubular neighbourhood of each MK6 carries  
*directed asymptotic transverse distance* from  $\frac{1}{2}$ M5 in MO9

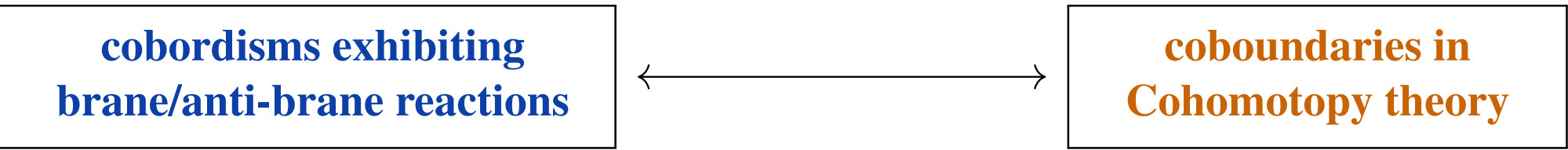
$$\Rightarrow \text{Cohomotopy charge: } X \xrightarrow[\text{directed asymptotic transverse distance from MK6 loci in MO9-planes}]{\text{Cohomotopy charge}} (\mathbb{R}^4)^{\text{cpt}} = S^4$$

spacetime manifold
4-sphere
Cohomotopy coefficient



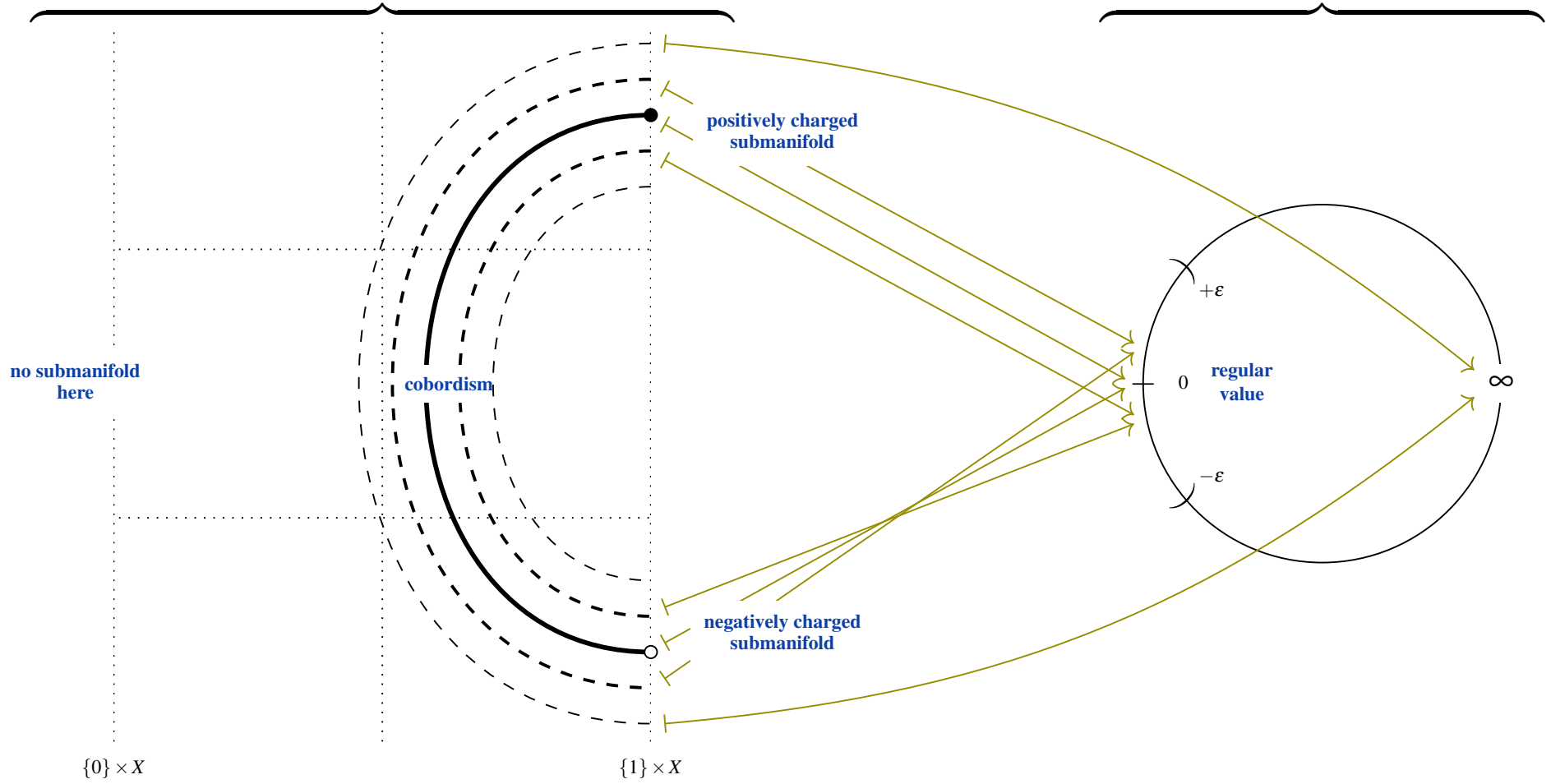
This construction and its reverse is Pontrjagin's construction ([Pon38], long before [Thom54]).

Under the above Pontrjagin construction one finds that:



$$[0, 1] \times X \xrightarrow{0 \simeq (-1) + (+1)} (\mathbb{R}^n)^{\text{cpt}} = \mathcal{S}^n$$

product space of interval with manifold      coboundary in Cohomotopy       $n$ -sphere Cohomotopy coefficient

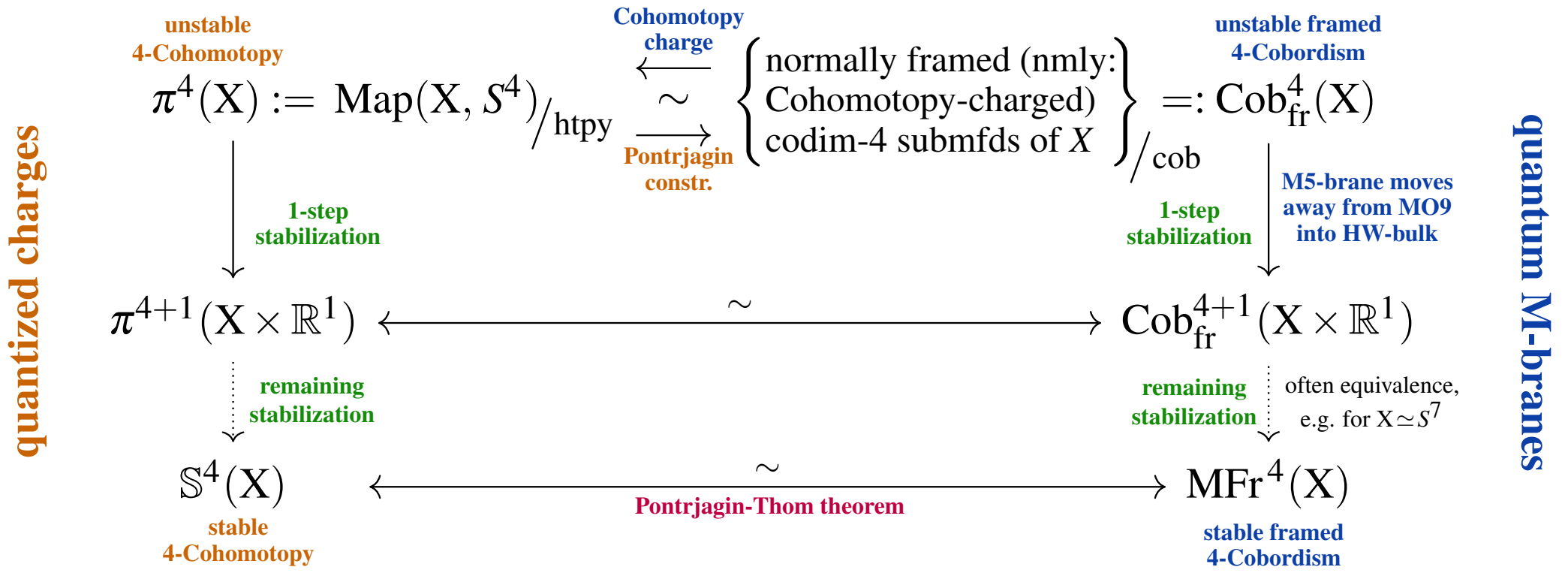


**Pontrjagin's theorem** says that

4-Cohomotopy is the conserved charge of these M-brane/anti-brane reactions in that the Cohomotopy charge map yields a *bijection* on cobordism classes:

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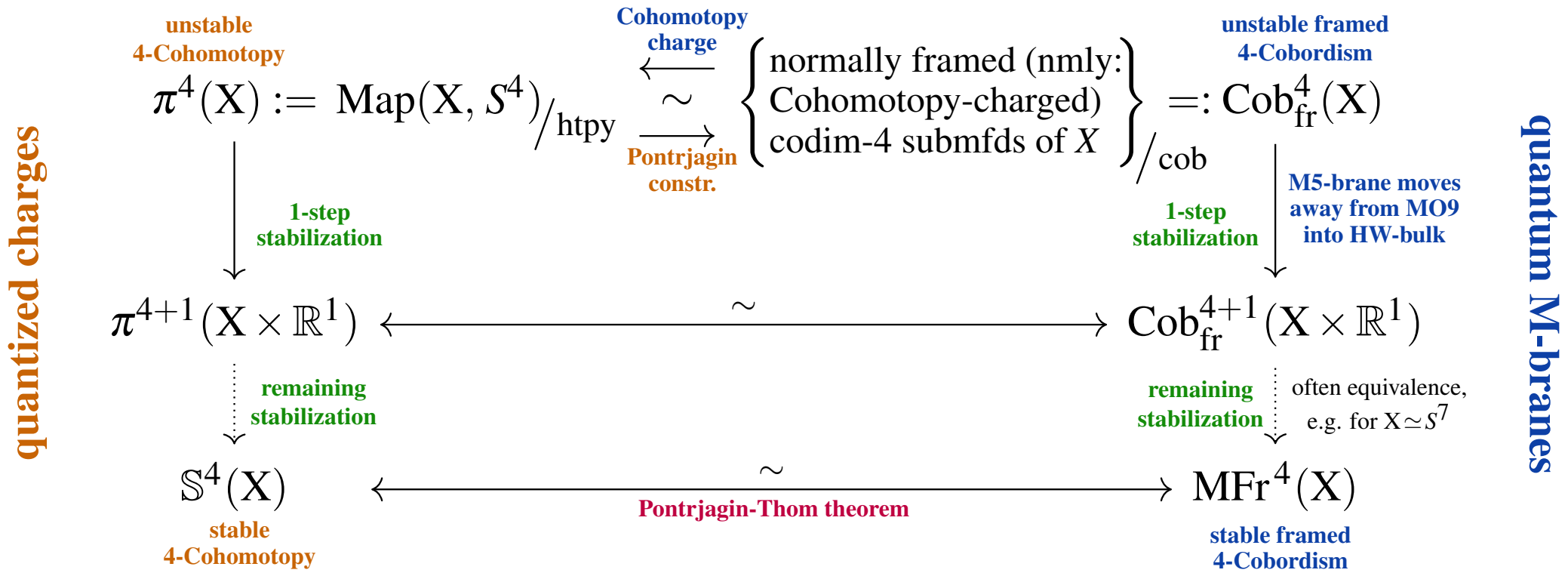
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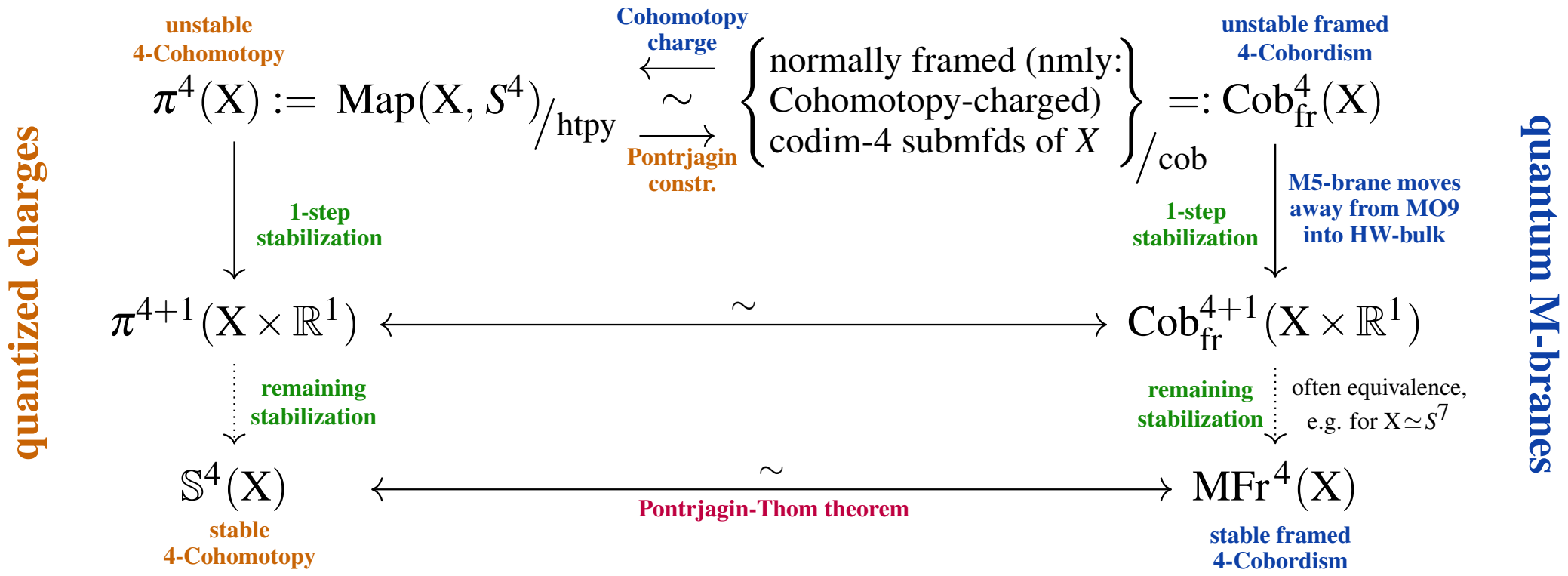
**Rem. 1.**

In particular this means that, in its stable = linearized approximation (cf above), Hypothesis H says equivalently that M-brane charge is quantized in stable framed Cobordism.

This is reminiscent of discussion in [McNamara & Vafa 19], see [SS21-MF, §4] for more.

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4-Cohomotopy is the conserved charge of these M-brane/anti-brane reactions in that the Cohomotopy charge map yields a *bijection* on cobordism classes:



**Rem. 2.**

$$\text{equivariant Cohom. cocycles } \text{Map}(X, S^4)^{G_{\text{ADE}}} \xrightarrow{\text{forget equivariance}} \text{Map}(X, S^4) \text{ plain Cohom. cocycles}$$

Via the forgetful map

$$\pi_{G_{\text{ADE}}}^4(X) \xrightarrow{e^*} \pi^4(X)$$

the PT theorem allows to *see* the quantum M-branes around orb-singularities:



# Equivariant Cohomology of flat orbifolds.

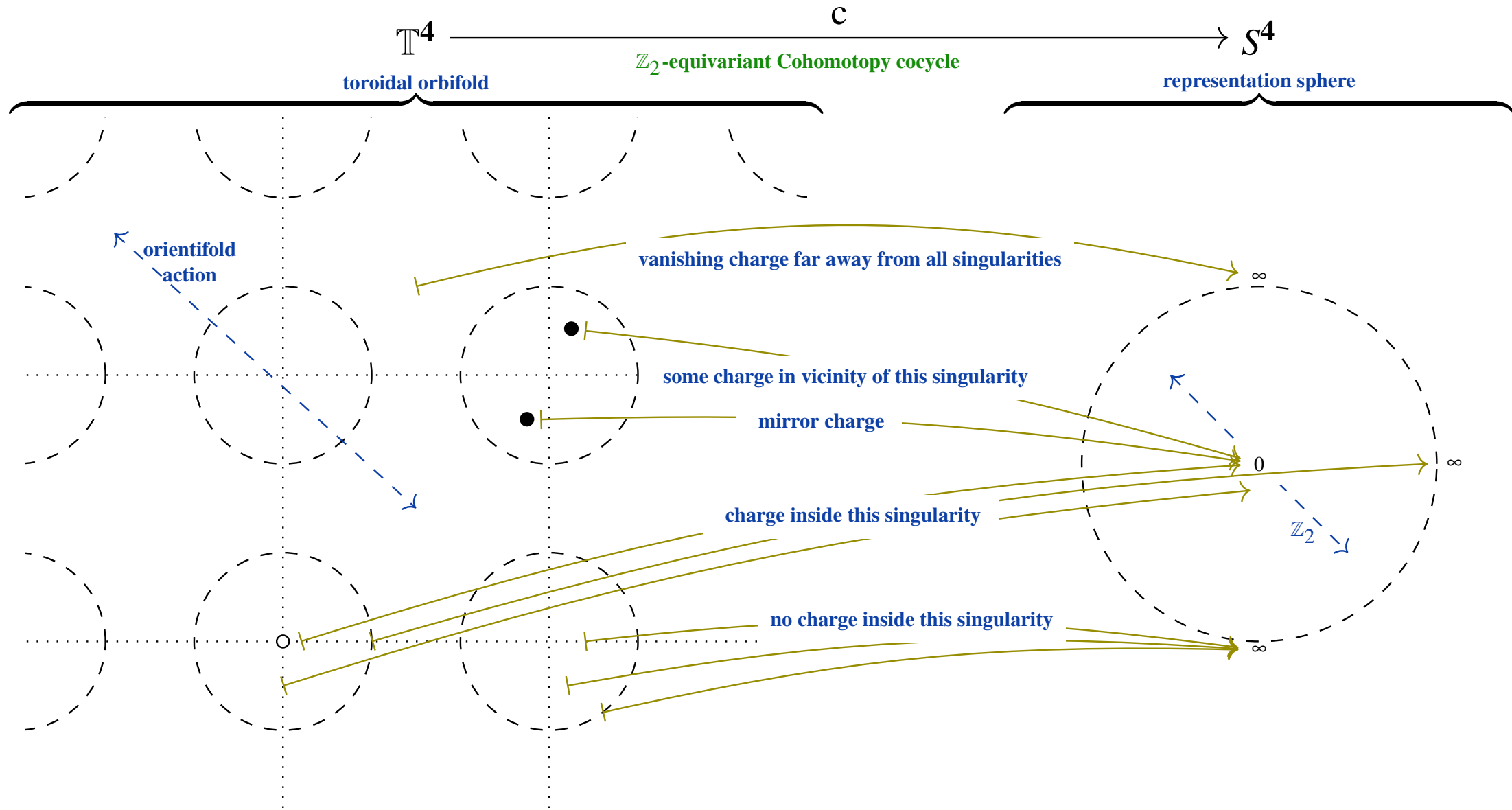
Thm. 3.17 in [SS19-TadCnc]

**Thm.:** Charges in  $G_{ADE}$ -equivariant 4-Cohomology of  $\mathbb{T}^4$  are labeled by:

1. a choice of charge  $\in \{0, -1\}$  *inside* each singularity;
2. an integer number of  $|G_{ADE}|$ -tuples of mirror unit charges.

**Proof.:** Use tom Dieck's equivariant Hopf degree theorem.

[tD79, §8.4]



## **Tadpole cancellation in Cohomotopy.**

[SS19-TadCnc]

**Informal idea** of tadpole cancellation in string-theory and in M-theory:

- 1) total brane charge in compact transversal space must vanish; and yet there
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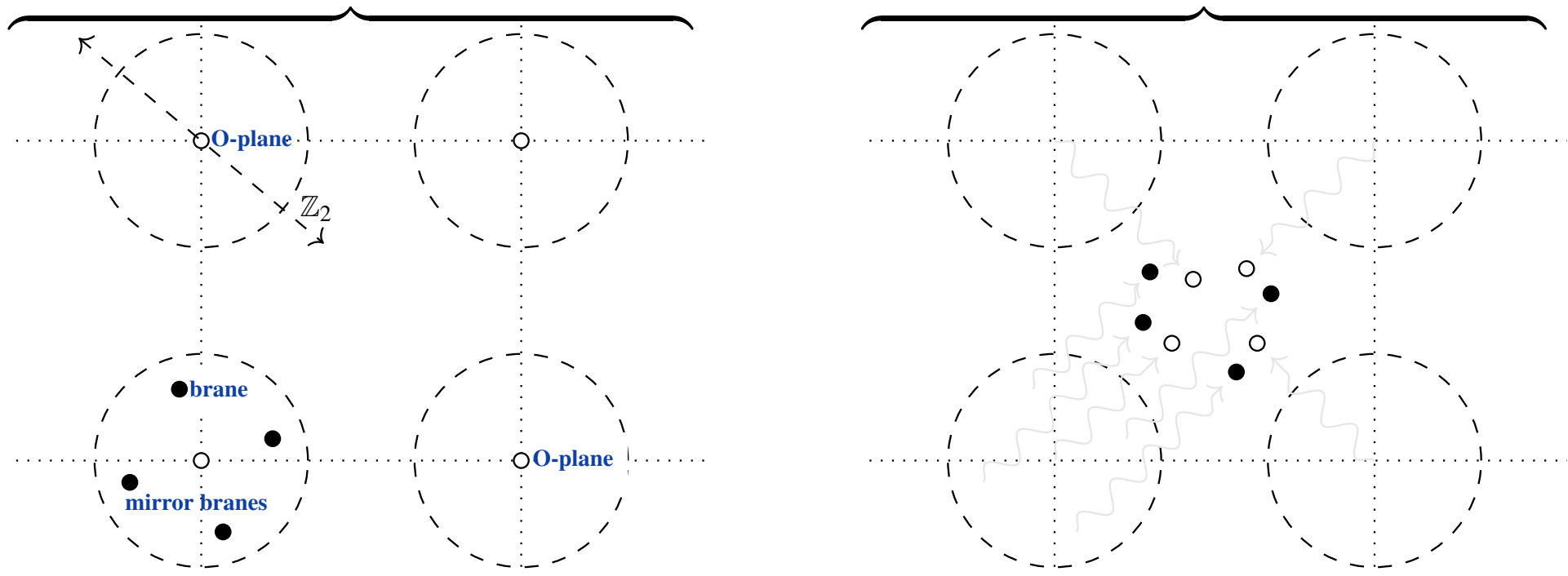
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**Formalization** in Cohomotopy theory according to Hypothesis H:

$$\begin{array}{ccc}
 \text{tadpole-free charges} & \pi_{G_{ADE}}^4(\mathbb{T}^4)_{\text{tot}=0} & \xrightarrow{\quad} \{0\} \\
 \text{fiber of tot-operation} & & \\
 & \downarrow & \downarrow \\
 \text{microscopic charges seen} & \pi_{G_{ADE}}^4(\mathbb{T}^4) & \xrightarrow[\text{sum up floating \& stuck charges inside orbifold}]{\text{tot}} \pi^4(\mathbb{T}^4) \\
 \text{in equivariant Cohomotopy} & & \text{non-abelian cohomology operation } (1 \longleftarrow G_{ADE})^* \\
 & \xrightarrow{2 \cdot 2 - 4 \cdot 1} & \xrightarrow{\quad} 0
 \end{array}$$

**Rem.** The condition  $\text{tot} = 0$  is naturally understood from *super-differential refinement* of Cohomotopy [SS19-TadCnc, (58)].



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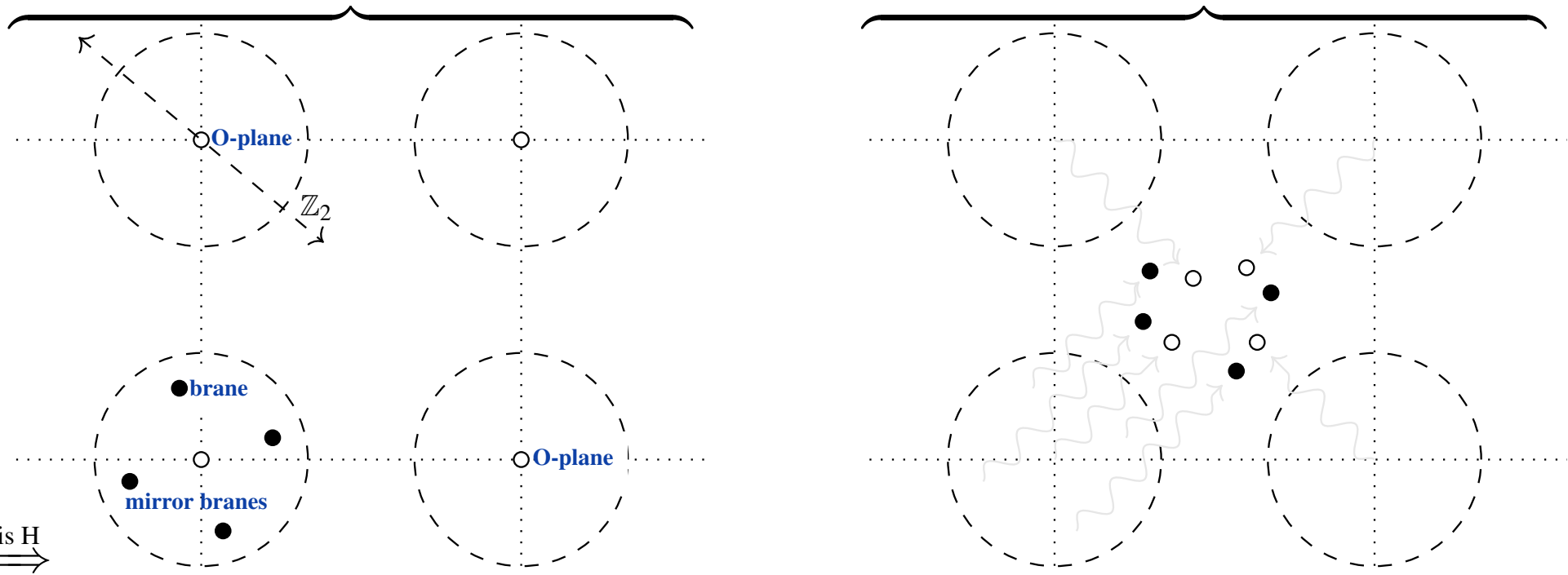
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**MO5-planes carry  $-\frac{1}{2}$ M5-brane charge.** Proves old conjecture: [DM95, §2][Wi 95, §3][Ho98, §2.1].

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Among the predictions in the other limit, of smooth but curved spacetimes X, stands out the shifted 4-flux quantization [FSS19-HypH, Prop. 3.13][FSS20-Char, §5.3]:

$$\pi^\tau(\mathbf{X}) \xrightarrow{\text{ch}} \left\{ \begin{array}{l} G_7, \in \Omega^\bullet(\mathbf{X}) \left| \begin{array}{l} dG_7 = -\frac{1}{2}G_4 \wedge G_4 + \dots \\ dG_4 = 0, [G_4 + \frac{1}{4}p_1(\omega)] \in H^4(\mathbf{X}; \mathbb{Z}) \end{array} \right. \end{array} \right\} / \sim_{\text{conc}}$$

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But various further consistency conditions on M-flux are expected, e.g.

Page charge quantization of  $G_7$ . Hypothesis H implies this, too: [FSS19-M5WZ, Thm. 4.8].

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*ab. cohomology*      *quantum states*  
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We identified [CSS21-Quant, Thm. 1.2 & §4] such states in top. sector of  $D6 \perp D8$ -branes and discovered [SS19-Quant] structures expected from DBI- and Hanany-Witten arguments.



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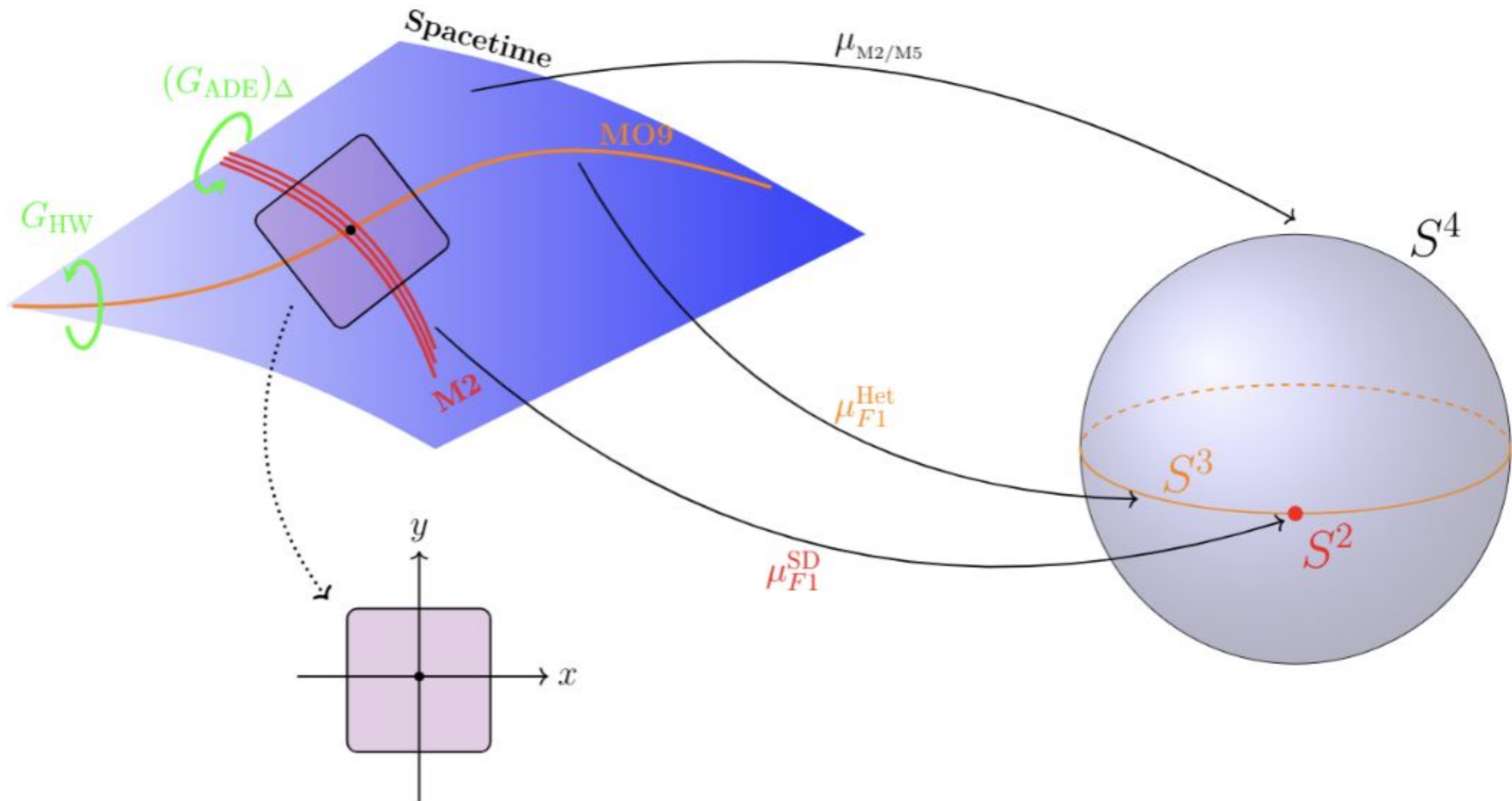
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So it looks promising...

these slides and further pointers are available at:

[ncatlab.org/schreiber/show/Proper+Orbifold+Cohomotopy+for+M-Theory](http://ncatlab.org/schreiber/show/Proper+Orbifold+Cohomotopy+for+M-Theory)



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