Urs Schreiber[a] on joint work with Hisham Sati[a]:

Introduction to *Hypothesis H*

lecture series at

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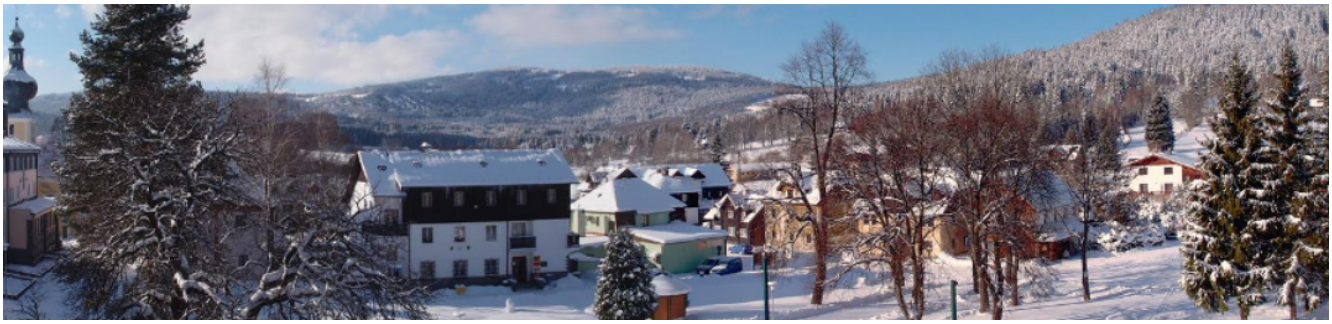

Session 1: **Non-Linear Flux Quantization in general** — (Context)

Session 2: **Flux Quantization on probe M5-Branes** — [1] — (Hypothesis H)

Session 3: **Topological Order on probe M5-Branes** — [2] — (Consequences)

Materials:

[1] *Flux Quantization*, Enzyclopedia of Mathematical Physics 2nd ed. **4** (2025) 281-324 [ncatlab.org/schreiber/show/Flux+Quantization]

[2] *Engineering of Anyons on M5 probes via Flux Quantization*, lecture notes (2025) [ncatlab.org/schreiber/show/Engineering+of+Anyons+on+M5-Probes]

[3] *The Character Map in Nonabelian Cohomology*, World Scientific (2023) [ncatlab.org/schreiber/show/The+Character+Map+in+Non-Abelian+Cohomology].

Abstract

The global (non-perturbative) definition of higher gauge fields requires *flux-quantization*. This is classical for electromagnetism (with Dirac quantization given by line bundles), and famous for RR-fields (whose quantization conjecturally is in topological K-theory). But when the higher Maxwell-type equations are non-linear in the fluxes (as they generically are in higher dimensional supergravity) then such Whitehead-generalized cohomology theories fail as flux-quantization laws, and what is needed are more general *non-abelian cohomology* theories (as in: non-abelian Poincaré duality).

I will first motivate and discuss the algebraic topology behind this statement, notably the generalization of the Chern character map from K-theory to nonabelian cohomology via methods of dg-algebraic rational homotopy theory, following our monograph [3].

Then I will turn to the example of 11d super-gravity, and explain how one viable flux quantization here (aka: a "model for the C-field") is given by twisted forms of Pontrjagin-Borsuk-Spanier "Co-Homotopy" theory, and how the choice of this law ("Hypothesis H") provably implies phenomena that have been conjectured to hold in the completion of 11d super-gravity by "M-theory" – such as the half-integral ($\lambda/2$-shifted) quantization of the 4-flux density.

Finally I will discuss how the corresponding cohomotopical flux-quantization of the "self-dual" tensor field, on M5-branes wrapping Seifert orbi-singularities, gives a novel, rigorous (and non-Lagrangian) derivation of anyonic topological order as seen in fractional quantum Hall systems, proving a conjecture by D. Gang et al.

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