Concretely, a differential form $\Psi$ on the configuration space which is $\omega_1$-twisted closed is equivalently an ordinary closed form $\widehat{\Psi}$ (51) on the universal cover of the configuration space of the following form (cf. [SS22-Any, (42)] following [FSV94, (20)][SV90, (2.1)], called the "master function" in [SlVa19, §2.1]):

$$\underbrace{\mathrm{d}\Psi + \omega_1(\vec{\mathrm{w}}, \kappa) \wedge \Psi = 0}_{\text{Twisted-closed wavefunction on configuration space}} \qquad \longleftrightarrow \qquad \underbrace{\widehat{\Psi}\big(\widehat{k}^1, \cdots, \widehat{k}^n\big)}_{\substack{\text{Equivariant closed wavefunction}\\ \text{on universal cover}}} = \prod_{1 \leq i < j \leq n} \underbrace{\big(\widehat{k}^i - \widehat{k}^j\big)^{2/\kappa}}_{\substack{\text{quanta-quanta (qq)}\\ \text{braiding phases}}} \prod_{\substack{0 \leq I \leq N \\ 1 \leq i \leq n}} \underbrace{\big(\widehat{k}^i - k_I\big)^{\mathrm{w}_I/\kappa}}_{\substack{\text{quanta-defect (qd)}\\ \text{braiding phases}}} \cdot \Psi\big(k^1, \cdots, k^i\big), \tag{62}$$

where $\widehat{k}^i$ denote coordinates on the universal cover, while $k^i$ denote the pullbacks of the corresponding coordinates (60) on the configuration space itself.

**Remark 3.11** (**Generalized Laughlin wavefunctions with mixed quanta-defect braiding phases).** The form (62) is just that of *generalized Laughlin wavefunctions* for anyons considered in [Hal84, (11)][NSSFS08, (89), (93)][La19, (3)], which generalize the original *Laughlin wavefunctions* [Lau83][MR91, §2.2] (review in [Gi04, §2.1]) to a situation with mixed quanta-defect braiding phases.

Hence for given $\kappa \in \mathbb{N}_+$ – determining the phase picked up by braiding any two anyonic quanta around each other – equation (61) parameterizes general quanta-defect braiding phases, subject only to the constrain that these come in integer multiples of *half* the quanta-quanta braiding phases. This curious constraint has its secret origin in the root lattice geometry of the Lie algebra $\mathfrak{su}_2$ and guarantees that the following crucial fact holds ([FSV94, Cor. 3.4.2, Rem. 3.4.3][SS22-Any, Prop. 2.17][9]):

> The complex de Rham cohomology of configuration space, twisted (58) by the "fictious vector potential" (61), naturally contains the space of $\mathfrak{su}_2$-*conformal blocks*, identified with the following Laughlin state (Rem. 3.11) Slater determinants (46) weighted by the canonical holomorphic volume form:

$$\begin{array}{ccc}
\underbrace{\mathrm{CnfBlck}_{\widehat{\mathfrak{su}_2}\,\kappa-2}}_{\substack{\mathfrak{su}_2\text{-affine}\\ \text{conformal blocks}\\ \text{at level } \kappa-2}}\Big(\underbrace{(k_I)_{I=0}^{N+1}}_{\substack{\text{on the Riemann sphere}\\ \text{with } N+1 \text{ punctures}}}, \underbrace{\Big\{(\mathrm{w}_I)_{I=0}^{N}, \mathrm{w}_{N+1} = \Big(\sum_{I=0}^{N} \mathrm{w}_I\Big) - n\Big\}}_{\text{of given weights}}\Big) & \longleftrightarrow & \underbrace{H_{\mathrm{dR}}^{n+\omega_1(\vec{\mathrm{w}},\kappa)}}_{\substack{\text{de Rham cohomology twisted}\\ \text{by "fictitious" vector potential}}}\Big(\underbrace{\underset{\{1,\cdots,n\}}{\mathrm{Conf}}\Big(\big(\widehat{\mathbb{T}}^2 \setminus S^1_a\big) \setminus \{k_I\}_{I=1}^{N}\Big)}_{\substack{\text{configuration space of}\\ n \text{ quanta among } N \text{ defects}}}; \mathbb{C}\Big) \\
\underbrace{f_{I_1} \cdots f_{I_n} \big|v_1^0 \cdots, v_N^0\big\rangle}_{\substack{\text{conformal block for } N \text{ punctures and } n \text{ insertions}\\ \text{(cf. [FSV94, 2.3.3, 2.3.6] [SS22-Any, Ex. 2.14])}}} & \longmapsto & \underbrace{\left[\det\left(\left(\frac{\mathrm{w}_{I_j}}{\kappa} \frac{1}{k^i - k_{I_j}}\right)_{i,j=1}^{n}\right) \mathrm{d}k^1 \wedge \cdots \wedge \mathrm{d}k^n\right]}_{\text{Slater determinant Laughlin state}}.
\end{array} \tag{63}$$

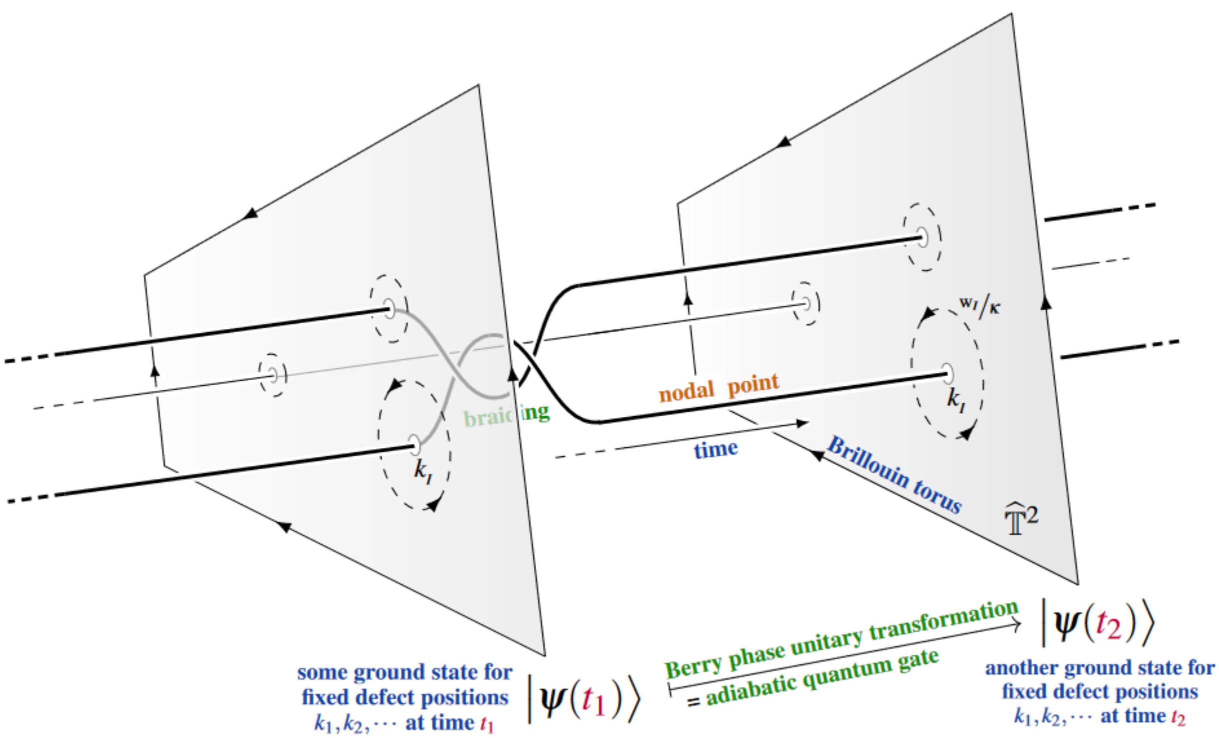


**Figure 1 – Adiabatic braid quantum gate**. Schematically indicated is the unitary transformation induced on the topologically ordered ground state (as discussed below in §3.3) of an effectively 2-dimensional topological semi-metal (as in §3.1) under adiabatic braiding (Rem. 1.1) of nodal points in the Brillouin torus (Rem. 3.9).