The homotopy data structure of topological quantum gates. With all these data structures in hand, we may conclude.

Definition 6.7 (Homotopy data structure of conformal blocks)**.** In specialization of Def. 5.16, we obtain this type:

$$
\left.\begin{array}{l}
\text{punctures} \quad \text{degree} \quad \text{shifted level} \\
N : \mathbb{N}_+, \;\; n : \mathbb{N}, \;\; \kappa : \mathbb{N}_{\geq 2} \\
\mathrm{w}_{(-)} : N \to \{0, \cdots, \kappa - 2\} \\
\text{weights}
\end{array}\right\}
\vdash
\bigg(\vec{z} \mapsto \Big[(t : \mathbf{B}\mathbb{C}^\times) \to \Big(\mathrm{fib}_{(t,\vec{z})}\big(\mathrm{pr}^{N+n}_N,\, \tau_{(\kappa,\mathrm{w}_\bullet)}\big) \to \mathbf{B}^n\big(\circlearrowleft_t \mathbb{C}_{\mathrm{udl}}\big) \Big) \Big]_0 \bigg) : \mathbf{B}\,\mathrm{PBr}(N) \to \mathrm{Type}
$$

where

$$
\begin{array}{lcccc}
(206) & \mathrm{pr}^{N+n}_N : & \mathbf{B}\,\mathrm{PBr}(N+n) & \longrightarrow & \mathbf{B}\,\mathrm{PBr}(N) \\
 & & \mathrm{pt} \overset{b_{I\,i}}{\circlearrowleft} & \mapsto & \mathrm{pt} \overset{\mathrm{e}}{\circlearrowleft} \\
 & & \mathrm{pt} \overset{b_{IJ}}{\circlearrowleft} & \mapsto & \mathrm{pt} \overset{b_{IJ}}{\circlearrowleft}
\end{array}
\qquad
\begin{array}{lcccc}
 & \tau_{(\kappa,\mathrm{w}_\bullet)} : & \mathbf{B}\,\mathrm{PBr}(N+n) & \longrightarrow & \mathbf{B}\mathbb{C}^\times \\
(53)\,(223) & & \mathrm{pt} \overset{b_{I\,i}}{\circlearrowleft} & \mapsto & \mathrm{pt} \overset{\exp(2\pi\mathrm{i}\frac{\mathrm{w}_I}{\kappa})}{\circlearrowleft} \\
 & & \mathrm{pt} \overset{b_{i\,j}}{\circlearrowleft} & \mapsto & \mathrm{pt} \overset{\exp(2\pi\mathrm{i}\frac{2}{\kappa})}{\circlearrowleft} \\
 & & \mathrm{pt} \overset{b_{IJ}}{\circlearrowleft} & \mapsto & \mathrm{pt} \overset{\exp(2\pi\mathrm{i}\frac{\mathrm{w}_I\mathrm{w}_J}{2\kappa})}{\circlearrowleft}
\end{array}
\qquad (224)
$$

Theorem 6.8 (**Topological quantum gates as homotopy data structure**)**.** *The semantics in the classical model topos* (107) *of the transport operation* (201) *in this type* (224) *is given by the monodromy of the Knizhnik-Zamolodchikov connection, on* $\widehat{\mathfrak{su}}_2{}^{\kappa-2}$*-conformal blocks (on the Riemann sphere with* $N+1$ *punctures weighted by* $(\mathrm{w}_I)_{I=1}^N$ *and* $\mathrm{w}_{N+1} = n + \sum_I \mathrm{w}_I$*).*

Proof. By Example 4.14 of Theorem 4.13, we are reduced to showing that the semantics of the type formation (224) equals the topological construction expressed by the formula (58). This follows by applying the syntax/semantics dictionary §5.1 iteratively to the sub-terms of (224), as shown in the following steps:

Syntax	$\overset{\S 5.1}{\longleftrightarrow}$	**Semantics**
$\vec{z} : \mathbf{B}\,\mathrm{PBr}(N),\; t : \mathbf{B}\mathbb{C}^\times \;\vdash$ $\mathrm{fib}_{(t,\vec{z})}\big(\mathrm{pr}^{N+n}_N,\, \tau_{(\kappa,\mathrm{w}_\bullet)}\big) \,:\, \mathrm{Type}$	(106) (5)	$\begin{array}{c} \underset{\{1,\cdots,N+n\}}{\mathrm{Conf}}(\mathbb{C}) \\ \big\downarrow{\scriptstyle (\mathrm{pr}^{N+n}_N,\, \tau_{(\kappa,\mathrm{w}_\bullet)})} \\ \underset{\{1,\cdots,N\}}{\mathrm{Conf}}(\mathbb{C}) \times B\mathbb{C}^\times \end{array}$
$\vec{z} : \mathbf{B}\,\mathrm{PBr}(N),\; t : \mathbf{B}\mathbb{C}^\times \;\vdash$ $\mathbf{B}^n\big(\circlearrowleft_t \mathbb{C}_{\mathrm{udl}}\big) \,:\, \mathrm{Type}$	(67) (175)	$p^*_{\underset{\{1,\cdots,N\}}{\mathrm{Conf}}} \left(\begin{array}{c} K(\mathbb{C}, n) \times_{\mathbb{C}^\times} E\mathbb{C}^\times \\ \big\downarrow \\ B\mathbb{C}^\times \end{array} \right)$
$\vec{z} : \mathbf{B}\,\mathrm{PBr}(N),\; t : \mathbf{B}\mathbb{C}^\times \;\vdash$ $\mathrm{fib}_{(t,\vec{z})}\big(\mathrm{pr}^{N+n}_N,\, \tau_{(\kappa,\mathrm{w}_\bullet)}\big) \to \mathbf{B}^n\big(\circlearrowleft_t \mathbb{C}_{\mathrm{udl}}\big) \,:\, \mathrm{Type}$	(66)	$\mathrm{Map}\left(\begin{array}{c} \underset{\{1,\cdots,N+n\}}{\mathrm{Conf}}(\mathbb{C}) \\ \big\downarrow{\scriptstyle (\mathrm{pr}^{N+n}_N,\, \tau_{(\kappa,\mathrm{w}_\bullet)})} \\ \underset{\{1,\cdots,N\}}{\mathrm{Conf}}(\mathbb{C}) \times B\mathbb{C}^\times \end{array} ,\; p^*_{\underset{\{1,\cdots,N\}}{\mathrm{Conf}}} \left(\begin{array}{c} K(\mathbb{C}, n) \times_{\mathbb{C}^\times} E\mathbb{C}^\times \\ \big\downarrow \\ B\mathbb{C}^\times \end{array} \right) \right)$
$\vec{z} : \mathbf{B}\,\mathrm{PBr}(N) \;\vdash$ $(t : \mathbf{B}\mathbb{C}^\times) \to T_{\vec{z},t} \,:\, \mathrm{Type}$	(67) (33)	$\big(\mathrm{id}_{\underset{\{1,\cdots,N\}}{\mathrm{Conf}}} \times P_{B\mathbb{C}^\times}\big)_* \begin{array}{c} T \\ \big\downarrow \\ \underset{\{1,\cdots,N\}}{\mathrm{Conf}}(\mathbb{C}) \times B\mathbb{C}^\times \end{array}$
$\vec{z} : \mathbf{B}\,\mathrm{PBr}(N),\; t : \mathbf{B}\mathbb{C}^\times \;\vdash$ $\big[(t : \mathbf{B}\mathbb{C}^\times) \to T_{\vec{z},t} \,:\, \mathrm{Type}\big]_0$	(129)	$\pi_{0/\underset{\{1,\cdots,N\}}{\mathrm{Conf}}(\mathbb{C})} \left(\big(\mathrm{id}_{\underset{\{1,\cdots,N\}}{\mathrm{Conf}}} \times P_{B\mathbb{C}^\times}\big)_* \begin{array}{c} T \\ \big\downarrow \\ \underset{\{1,\cdots,N\}}{\mathrm{Conf}}(\mathbb{C}) \times B\mathbb{C}^\times \end{array} \right)$
Homotopy type structure of Def. 6.7 specializing Def. 5.16	$\leftrightarrow$	Fibration of conformal blocks (58) via Thm. 4.13 & Ex. 4.14

□