The basic ideas for how to do this date back 10 years:

arXiv > math-ph > arXiv:1402.7041

Mathematical Physics

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Quantization via Linear homotopy types

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3. *A spectrum object E in $\mathbf{H}$ is a collection of objects $(E_n \in \mathbf{H}^{*/})_{n\in\mathbb{Z}}$ equipped with equivalences $(E_n \xrightarrow{\simeq} \Omega E_{n+1})$.*

4. *The obvious maps of diagrams between spectrum objects in $\mathbf{H}_{/X}$ as $X \in \mathbf{H}$ ranges form the* tangent ∞-category $T\mathbf{H}$ *of* $\mathbf{H}$. [Joyal 08]

Proposition 2.5. *$T\mathbf{H}$ is itself an ∞-topos. The types in $T_X\mathbf{H}$ are stable homotopy types in $\mathbf{H}_{/X}$.* [Joyal 08]

Example 2.6. *For $\mathbf{H} = \infty\mathrm{Grpd}$ then a homotopy type in $T_*\mathbf{H}$ is equivalently an ordinary spectrum and for any type $X \in \infty\mathrm{Grpd} \hookrightarrow T\infty\mathrm{Grpd}$ then*

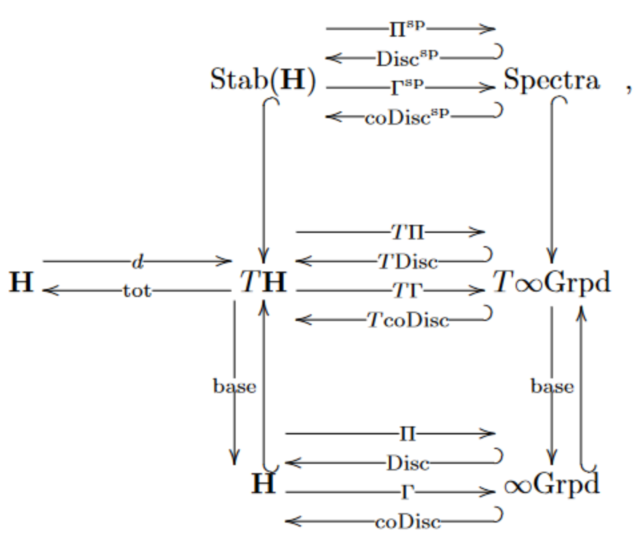
$$E^\bullet(X) \simeq (X \to E)$$

is the E-cohomology spectrum of X. For $\tau \in T_X\mathbf{H}$ a bundle of spectra whose fibers are equivalent to E, then

$$E^{\bullet+\tau}(X) \simeq (X \to \tau)$$

is the τ-twisted E-cohomology spectrum of X [ABGHR 14].

Proposition 4.1.9. *If $\mathbf{H}$ is an ∞-topos which is cohesive, def. 3.4.1, then its tangent ∞-topos $T\mathbf{H}$, prop. 4.1.8, is cohesive over $T\infty\mathrm{Grpd}$ and infinitesimally cohesive def. 3.5, over $\mathbf{H}$. Moreover, the cohesive structure maps fit into a diagram of the form*



0.1 Motivation

We lay out an approach to a joint solution of the following open problems:

(I) The open problem of reliable quantum computing. While the hopes associated with quantum computing (Lit. 1.1) are hard to overstate, experts are well-aware[1] that currently existing hard- and soft-ware paradigms are unlikely to support the desired heavy-duty quantum computations beyond toy examples. The two fundamental open problems that the field still faces are both rooted in the single most enigmatic and proverbial phenomenon of quantum physics: the *state collapse* or *decoherence* phenomenon (Lit. 1.2), whereby the peculiar non-classical properties of quantum systems on which rest the hopes of quantum computing are jeopardized by any measurement-like interaction of the system's environment. This means that scalably robust quantum computing requires:

- **(i) Topological hardware** (Lit. 1.3) given by topological quantum materials (Lit. 1.23) whose registry-states are protected by an "energy gap" from having *any* interaction with the environment below that range.
- **(ii) Verified software** (Lit. 1.4) with compile-time certificates of correctness — since the traditional run-time debugging of complex programs is impossible for quantum programs (causing collapse), while all the more needed due to the complexity and intransparency of gate-level quantum circuits.

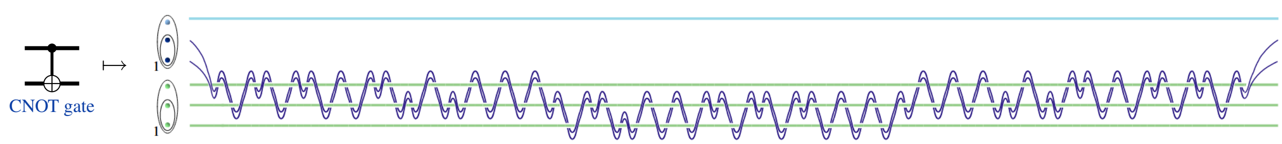
Both of these issues have been discussed separately, but the necessary combination has remained essentially untouched until [TQP]; one will need a quantum programing language (Lit. 1.5) which is

- **(iii) certifiable and topological-hardware-aware**, allowing the programmer to formally verify at compile-time the correctness not (just) of high-level quantum programs, but of quantum circuits consisting of the peculiar topological quantum gates that the topological quantum hardware actually provides.

For example, to state just the most immediate problem:

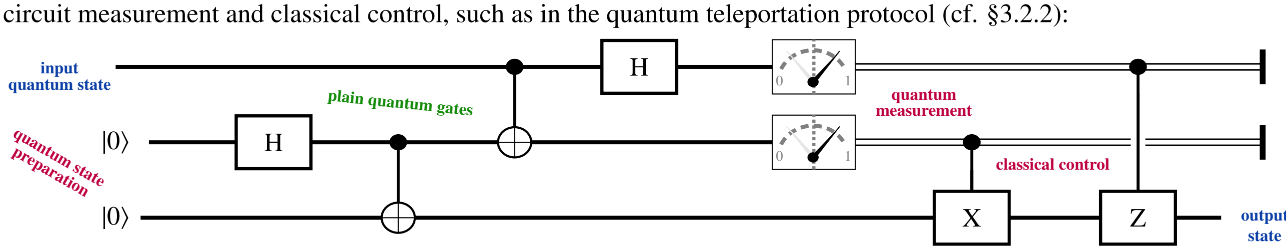
Topological quantum circuit compilation problem (Lit. 1.9).
Suppose a topologically ordered quantum material is finally developed which features su_2*-anyon states at level* ℓ*, and given any quantum circuit written in the usual QBit-basis, then the quantum compilation of this circuit onto the given hardware is the specification of a braid (an element of a braid group) such that the holonomy of the* su_2^ℓ *Knizhnik-Zamolodchikov connection along the corresponding path in the configuration space of defect points in the given quantum material may be conjugated onto the unitary operator to which the quantum circuit evaluates, within a specified accuracy.*

Here the relevant braids are humongous while having no recognizable resemblance to the quantum algorithm which they are executing; for instance, a single CNOT gate (17) may compile to the following braid [HZBS07, Fig. 15]:



Hence future quantum programmers will need (classical) computer assistance to compile their quantum programs onto topological hardware. To make that intricate process fail-safe to reliably run on precious scarce quantum resources, we need this computer algebra to be "aware" of the system specification and to certify its own correctness relative to this specification. And this is just for the simplest case of no classical control. The general problem is harder still:

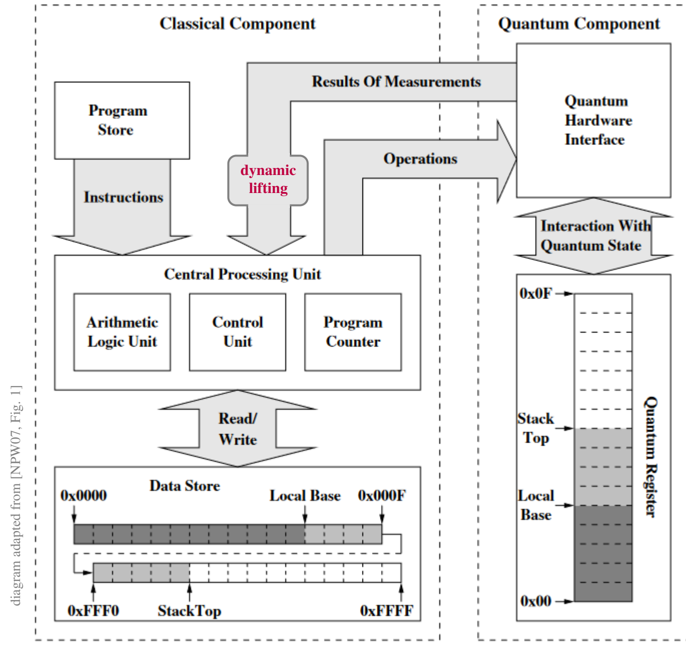
The problem of certifying classical control. Even the most elementary quantum information protocols involve mid-circuit measurement and classical control, such as in the quantum teleportation protocol (cf. §3.2.2):



[1][Sau17]: "small machines are unlikely to uncover truly macroscopic quantum phenomena, which have no classical analogs. This will likely require a scalable approach to quantum computation. [...] based on [...] topological quantum computation (TQC) as envisioned by Alexei Kitaev and Michael Freedman [...] The central idea of TQC is to encode qubits into states of topological phases of matter. Qubits encoded in such states are expected to be topologically protected, or robust, against the 'prying eyes' of the environment, which are believed to be the bane of conventional quantum computation."

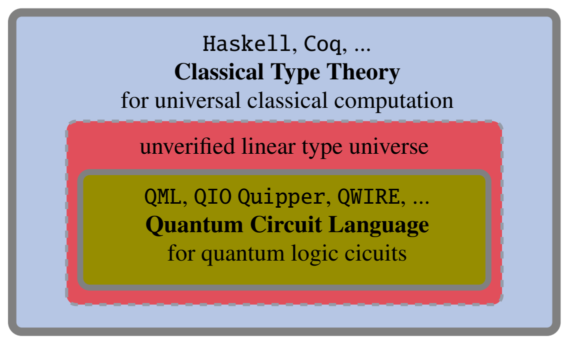
[DS22]: "The qubit systems we have today are a tremendous scientific achievement, but they take us no closer to having a quantum computer that can solve a problem that anybody cares about. [...] What is missing is the breakthrough [...] bypassing quantum error correction by using far-more-stable qubits, in an approach called topological quantum computing."

More importantly, beyond the currently available NISQ paradigm (Lit. 1.10), serious quantum computation is expected (Lit. 1.11) to involve a perpetual loop of classical control operations on the quantum computer (*hybrid* classical/quantum computation). These are predominantly for quantum error correction (§3.2.3) but also for purposes such as repeat-until-success gates – where subsequent quantum circuit execution is classically conditioned on run-time quantum measurement results – also called "dynamic lifting" (Lit. 1.11, namely of quantum measurement results into the classical data register). This is schematically indicated on the right.
Last not least, for probabilistic analysis of such hybrid processes the machine state is to be modeled as a *mixed* classical/quantum probabilistic state (Lit. 1.12).



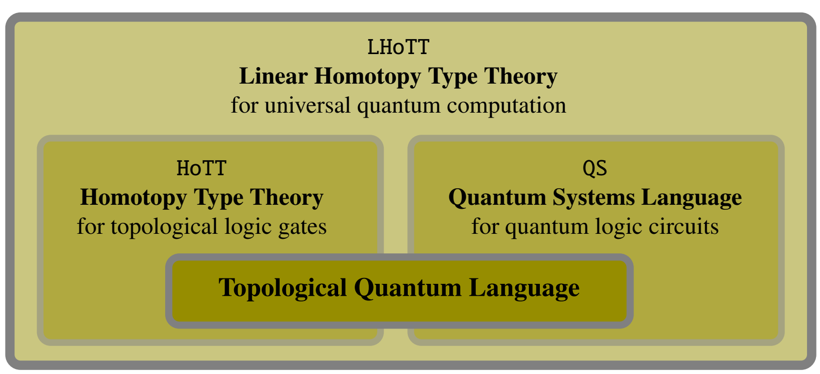
Hence for reliable heavy-duty quantum computation we need a certification language that knows about classical data types *and* about linear/quantum data types *and* their *dependency* on classical data. This had been lacking:

The problem of embedded quantum languages. Namely, for previous lack of a *universal* quantum programming language, existing quantum circuit languages are embedded into *classical* host languages (Lit. 1.5) which do not have native support for linear types (cf. Lit. 1.4) nor for classical control of quantum circuits. For instance, basic protocol schemes such as quantum teleportation (§3.2.2), quantum error correction (§3.2.3) or repeat-until-success gates remain unverifiable with previous technology.



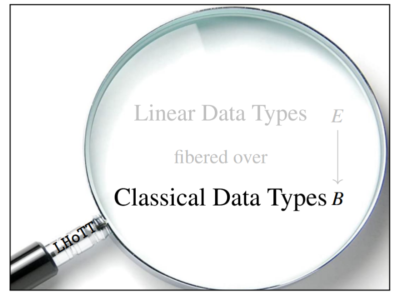
Solution by Linear Homotopy Type Theory. We argue here, as announced in [Sch22], that the novel type theory `LHoTT` (Lit. 1.8) recently developed in [Ri22a] (anticipated in [Sch14a]) in extension of the classical language scheme `HoTT` (Lit. 1.7) serves as the missing universal quantum programming/certification language.
Our claim in [QS] is that `LHoTT`:

- Solves the old problem of constructing combined classical/linear type theories (cf. Lit. 1.4).
- Provides existing quantum programming languages like `Quipper` with a certification mechanism [Ri23].
- Natively supports quantum effects such as dynamic lifting of run-time quantum measurement (§2).
- Natively supports verification of realistic topological quantum gates [TQP].



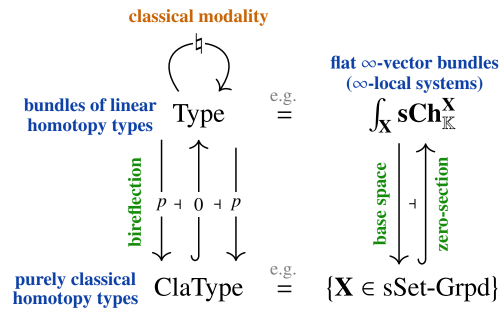
We argue that this makes `LHoTT` the first comprehensive paradigm for serious quantum programming beyond the NISQ area. Here we describe a domain-specific language embeddable into `LHoTT` to bring this out: *Quantum Systems Language* (`QS`, §3), based on a system of monadic effects which are definable (by admissible inference rules) in `LHoTT` (§2, surveyed below in §0.2).

Concretely, LHoTT enhances the syntactic rules of classical HoTT by further type formations which serve to exhibit every (homotopy) type E of the language as secretly consisting of an underlying classical (intuitionistic) base type $B \equiv \natural E$ equipped, in a precise sense, with a microscopic (infinitesimal) halo of linear/quantum data. As such, LHoTT may neatly be thought of as the formal logical expression of a microscope that resolves quantum aspects on structures that macroscopically appear classical. This way LHoTT embeds quantum logic into classical logic in a way reminiscent of Bohr's famous dictum[2] that all quantum phenomena must be expressible in classical language.



Quantum halos. Formally this is achieved by adjoining to classical HoTT an *ambidextrous* modal operator $\natural$ [RFL21] (an *infinitesimal cohesive modality* [Sch13, Def. 3.4.12, Prop. 4.1.9]), whose modal types (Lit. 1.14) are the *purely classical* (ordinary) homotopy types, embedded *bi-reflectively* (157) among all data types (see §2.1):

The presence of the $\natural$-modality exhibits general types E : Type as microscopic/infinitesimal *halos* around their underlying purely classical type $\natural E$: ClaType. It is a profound fact (146) of ∞-topos theory that models for such *infinitesimal cohesion* (see Lit. 1.21) are provided by parameterized module spectra, in particular by flat ∞-vector bundles ("∞-local systems", see [EoS]) which, in their 0-sector (Rem. 1.22), accommodate quantum circuit semantics (cf. §2.4) in indexed sets of vector spaces (cf. §2.1) such as known from the Proto-Quipper quantum language (Lit. 1.5).



(1)

Motivic Yoga. LHoTT witnesses these quantum halos as *linear types* (24) equipped with a closed tensor product $\otimes$ and compatible base change operations which satisfy the rules of Grothendieck's "motivic yoga of six operations" in Wirthmüller style (Def. 2.18, cf. [Ri22a, §2.4][EoS, §3.3]). It is this "motivic" structure from which the structure of quantum physics derives, as originally observed in [Sch14a] and here brought out in §2.1.

Linear/Quantum Data Types

Characteristic Property	**1.** Their cartesian product blends into the co-product:	**2.** A tensor product appears & distributes over direct sum	**3.** A linear function type appears adjoint to tensor
Symbol	$\bigoplus$ direct sum	$\otimes$ tensor product	$\multimap$ linear function type
Formula (for W : ClaType$^{\mathrm{fin}}$)	cart. product $\prod_W \mathcal{H}_w \simeq \oplus \mathcal{H}_w \simeq \coprod_W \mathcal{H}_w$ co-product; direct sum	$\mathscr{V} \otimes \big(\bigoplus_{w:W} \mathcal{H}_w \big) \simeq \bigoplus_{w:W} (\mathscr{V} \otimes \mathcal{H}_w)$	$(\mathscr{V} \otimes \mathcal{H}) \multimap \mathscr{K} \simeq \mathscr{V} \multimap (\mathcal{H} \multimap \mathscr{K})$
AlgTop Jargon	biproduct, stability, ambidexterity	Frobenius reciprocity	mapping spectrum
		Grothendieck's Motivic Yoga of 6 oper. (Wirthmüller form)	
Linear Logic	additive disjunction	multiplicative conjunction	linear implication
Physics Meaning	parallel quantum systems	compound quantum systems	qRAM systems

***H*$\mathbb{C}$-Linear quantum theory.** In this scheme, conventional quantum information theory happens in the $\mathbb{C}$-linear form of linear homotopy theory (details in [EoS]) where parameterized $H\mathbb{C}$-module spectra are equivalent to *flat ∞-bundles of chain complexes*, also known as *∞-local systems*. Here the higher structure of chain complexes serves to capture topological quantum effects [TQP], but in the 0-sector (Rem. 1.22) these are just set-indexed complex vector spaces of the form familiar from the categorical semantics of the quantum language Quipper, this is what we discuss in detail §2.1 . But since all our quantum effects are constructed monadically (§2) relying just on the abstract Motivic Yoga, they apply at once to unrestricted (stable) homotopy types, providing a homotopy-theoretic form of quantum mechanics suitable for the discussion of "topological quantum effects" as in [TQP].

[2][Bohr1949, pp. 209]: "however far the phenomena transcend the scope of classical physical explanation, the account of all evidence must be expressed in classical terms". For background and commentary see also [Sche73, p. 24].