HIGHER THETA FUNCTIONS AND HIGHER CS/WZW HOLOGRAPHY

Dr Urs Schreiber (ncatlab.org/nlab/show/Urs+Schreiber)

PART I: TRACK RECORD – CONTRIBUTIONS DIRECTLY RELEVANT TO THE PROPOSAL

Positions and Qualifications. from 2015 Research Fellow at the Institute of Mathematics at the Czech Academy of the Sciences • 2015 Visiting Professor, E Čech Inst for Algebra, Geometry and Physics, Charles Univ Prague • 2012–2014 Postdoctoral Research Fellow, Univ of Nijmegen • 2009–2012 Postdoctoral Research Fellow, Univ of Utrecht • 2011 Habilitation, Univ Hamburg • Summer 2008 Research Fellow, Hausdorff Institute Bonn • 2005–2010 Postdoctoral Research Fellow, Univ of Hamburg • 2005 PhD, Univ of Essen • 2002 Diplom (MSc by research) in Physics, Univ of Essen

Current Areas of Research. Differential Geometry • Homotopy Theory • Mathematical Quantum Field Theory • Mathematical String Theory

Track Record. In my Habilitation thesis [S25], I have initiated a new mathematical approach to the modern *higher* (homotopy theoretic, *derived*) version of differential geometry and its application to geometric problems in quantum field theory (QFT). I have delivered many lectures on this approach at international conferences [S14–S24], and published a wide range of applications with various collaborators (a review exposition is in [S7]). The research proposed here will advance this new approach to a whole new level. The following highlights some of my contributions (with various co-authors) in my research areas that are directly relevant to the proposed project; introduction and background is below in Part II. Specifically,

I.1) I have lifted [S25] *axiomatic cohesion* [39] to higher topos theory [41], found models of practical interest [S13, S8, S23], and showed that this cohesive homotopy theory axiomatizes differential cohomology. This is the mathematical framework for all aspects of abelian [18] and nonabelian/twisted [S14, S13] higher gauge field theory. This axiomatics has been used [12] to solve a conjecture of Simons–Sullivan (reviewed in [S20]) and to clarify the structure of equivariant homotopy theory [51]. The axioms are strikingly elegant, contrasting with the complexity of the theory that they encode. This allowed the construction of new models of differential K-theory [12] and twisted differential cohomology [11] crucial for mathematical QFT and string theory [16]. It also enabled me [S28] to embed the axioms into the new foundations called *homotopy type theory* [58], see [6, S19] for more on *cohesive homotopy type theory*. This gives a practicable way to apply computer-aided formal proof to all aspects of this proposal; first steps in this direction were already carried out [8].

I.2) I have established [S10] geometric higher principal bundle (higher gerbe) theory and the classification theorem by higher nonabelian cohomology represented by $\mathbf{B}G$ (stacky deloopings). This result was used to solve the Beilinson–Drinfeld problem in Tate vector bundles via algebraic K-theory [52]. It was also used to generalize the nonabelian Hodge theorem of Simpson to twisted bundles [25].

I.3) I have proved [S25] that for G a Lie group then homotopy classes of maps of higher stacks $\mathbf{L} : \mathbf{B}G \to \mathbf{B}^n A$ (for $A = \underline{\mathbb{R}}$, $\mathbb{Z}$, $\underline{\mathrm{U}}(1)$) correspond to Segal–Brylinski's refined Lie group cohomology of G with coefficients in A. Hence, for G a compact Lie group, the homotopy classes of maps $\mathbf{L} : \mathbf{B}G \to \mathbf{B}^n U(1)$ are in bijection with the *levels* in Chern–Simons (CS) gauge theory. This is a basis for discussion of fully local CS theory in this proposal. It influenced similar analysis in [59].

I.4) I have shown [S9] that the higher Lie integration [30] has a differential refinement that reads in transgressive cocycles of strong-homotopy Lie algebras and Lie-integrates them to homomorphisms of moduli stacks $\mathbf{B}G_{\mathrm{conn}} \to \mathbf{B}^{n+1}U(1)_{\mathrm{conn}}$ of higher principal *connections* (whence the subscript). This pioneered the use of the homotopy theory of simplicial sheaves in higher Chern–Weil theory; complementary results were later given in [22]. All of [S1, S4, S6–S8] builds on this construction.

I.5) I have shown [S6–S8] that these differential refinements $\mathbf{L}_{\mathrm{conn}} : \mathbf{B}G_{\mathrm{conn}} \to \mathbf{B}^n U(1)_{\mathrm{conn}}$ are the de-transgressed *fully local CS Lagrangians* i.e. their *transgression* to moduli stacks of fields on any n-dimensional closed manifold Σ is the actual CS action. The intermediate transgression to $(n-1)$-dimensional Σ is the CS pre-quantum line bundle—which is the *theta line bundle* of pivotal interest in the present proposal—and the transgression to $(n-2)$-dimensional Σ is the higher Wess–Zumino–Witten (WZW) gerbe. This encodes the super p-branes in string theory as higher super-WZW models

[S8], anticipated in [S12,35]. This also includes the 6d field theory of the M5-brane of concern here. These towers of (de-)transgressions of higher CS Lagrangians motivate the present proposal.

I.6) I have shown [S13] that the homotopy fiber products of the above $\mathbf{L}_{\mathrm{conn}}$ yield the higher moduli stacks of *Green–Schwarz anomaly-free* heterotic string configurations, and yield [S5] a refinement of the 11d supergravity C-field to higher stacks, necessary for the fully local description of the nonabelian 7d CS Lagrangian that I constructed in [S4]. This is a key structure in the present proposal.

I.7) I have shown [S1] that differential integration, when applied to any symplectic Lie n-algebroid, yields a full de-transgression of the Alexandrov–Kontsevich–Schwarz–Zaboronsky (AKSZ) action functionals. The plain AKSZ functionals have more recently been re-popularized [50]. I have shown [S2,10] that applied to Poisson Lie algebroids the resulting 2d Poisson-Chern-Simons theory has as moduli stack of fields a differential refinement of the *symplectic groupoid*.

I.8) I have shown [S27] that the *geometric cohomological boundary quantization* of this 2d Poisson–Chern–Simons theory gives geometric Spin^c quantization of compact Poisson manifolds. This is the content of the theses [10,49] written under my supervision. This is a non-perturbative analog of the celebrated deformation quantization of Poisson manifolds via the perturbative Poisson sigma-model [14].

I.9) I have shown [S2,S3] that these fully localized CS functionals, when viewed as objects in the cohesive slice homotopy theory over $\mathbf{B}^n U(1)_{\mathrm{conn}}$, are higher pre-quantum line bundles whose higher automorphism groups are the higher analogs of Kostant–Souriau central extensions including higher Heisenberg groups and whose strong-homotopy Lie algebras are higher Poisson bracket algebras of local observables/conserved currents. Further aspects of this have recently been studied [29]. Moreover, I have shown [S26,S18] that this provides a comprehensive formalization and pre-quantum refinement of de Donder–Weyl localized classical Hamilton–Lagrange–Jacobi field theory .

I.10) I have discussed various aspects of a theory of cohomological geometric quantization of such data by Gysin maps in twisted generalized cohomology theories [S27]. Related discussion is to appear in [33].

Altogether, these results establish a fully localized/de-transgressed formalization of classical and pre-quantum higher-dimensional CS-type Lagrangian field theory—geometrically (cohesively) refining the higher-dimensional but geometrically discrete Dijkgraaf–Witten-type theory [23]—and of first aspects of cohomological geometric quantization of such data. This now allows one to study the corresponding higher analog of *theta functions* and of higher *CS/WZW holography* (explained in Part II) with the tools of cohesive homotopy theory [S25] and with the proof of the cobordism hypothesis [42,29].

PART II: DESCRIPTION OF PROPOSED RESEARCH AND ITS CONTEXT

Introduction. Quantum field theory (QFT) and string theory are notorious for involving structures just outside the reach of available mathematics and hence, for providing deep inspiration for new mathematics (recent developments are compiled in [S11]). The special cases of *conformal* and *topological* QFTs have always been more amenable to mathematical investigation [5,54,24], but they are often regarded as mere toy examples. Progress in recent years, both in mathematics and theoretical physics, has changed this picture:

II.1) The recent development of homotopy theory and the formalization and proof of the cobordism hypothesis [42] (i.e. classifying fully local topological QFT via higher monoidal category theory, see [9]) shows that the axioms for topological QFT originally found by Atiyah & Segal [5,54] are, in their refined *local* form, much more deeply rooted in the bedrock of mathematics than previously thought. In a sense, [42] shows that local topological quantum field theory is the unstabilized generalization of the J-homomorphism in stable homotopy theory. This gives the axioms a deep mathematical relevance quite independent of any physics, witnessed by recent developments such as [17].

II.2) The development of the holographic principle [43] in physics highlights, in view of its analysis in [62,28], that topological QFTs (inside string/supergravity theories) serve to induce actual physical field theories *on their holographic boundaries*, and, in fact, that chiral/self-dual higher gauge fields are only definable this way in the first place [61,46]. In terms of actual mathematics, this is nicely exhibited by the classification result [24] which constructs (modulo the pertinent identification of spaces of conformal blocks) rational 2d conformal field theories (CFTs) in a holographic fashion from 3d Chern–Simons (CS) theory, see [37] for review amplifying this perspective. Moreover, [61,64,65] argues that via a chain of reductions and equivalences, there is a whole cascade of non-topological field theories descending from a single topological one. Notably, the prime example of all QFTs since the 1950s—4d Yang–Mills theory—is supposed to be usefully understood as the compactification of a 6d CFT holographically defined by 7d CS theory (sitting inside some supergravity completion). The quest for understanding this 6d theory has recently been attracting much attention [44,21], but the fact that its only actual definition is via holography involving a 7d CS theory [62,S4] should be kept in mind (see e.g. [45]).

On the other hand, while in physics one routinely thinks of QFTs as induced from geometric data—namely via *quantization* of Lagrangians—a reflection of this geometric quantization process (a modern review is in [10]) in the refined local axiomatics of the cobordism hypothesis has been missing. But much of the interesting structure resides precisely here.

A.1. Research Hypothesis and Objectives

Major Aims. The broad goal of the proposed research program is to find, formulate, and study the refinement of the all-important mathematics of geometric quantization of topological field theory to the fully local (*extended*, *multi-tiered*) setup of the cobordism hypothesis. I have already provided some of the foundations [S25–S27] (following [S14] and inspired by a sketch in [23]). In these references, I presented a formulation of *local pre-QFT* via combining the cohesive homotopy theory which I developed with the proof of the cobordism hypothesis. Rather recently, aspects of this have been developed further in [29,19,33] but nevertheless, many open problems remain.

Research Hypothesis. The research hypothesis of the project is proof of a new claim thats was previously sketched in [S25] but waiting to be spelled out. Concretely, this claim states that by means of [42,29], the local cohesive pre-QFT induced by a fully local higher cohesive CS functional $\mathbf{L} : \mathbf{B}G \to \mathbf{B}^n U(1)$, as in Track Record I.5), automatically sends a closed manifold Σ of codimension-one to the pre-quantum/theta line bundle $\Theta : \mathbf{Loc}_G(\Sigma) \to \mathbf{B}U(1)$ of the higher CS-type field theory given by the transgression of $\mathbf{L}$ to the higher moduli stack of G-local systems on Σ. As indicated below in Background, this yields a striking confluence of a number of deep and time-honored phenomena and at the same time provides their generalization to higher dimensions and to various flavors (super-, derived-, arithmetic-,...) of geometry.

Objectives. Therefore, this hypothesis opens the door to a systematic definition and study of the following four subjects, lifting their well-known low-dimensional counterparts. Specifically, I will develop and study:

Objective 1 – Higher Modular Functors	**Objective 2** – Higher Topological Modular Forms
Objective 3 – Higher Langlands correspondence	**Objective 4** – Higher Equiv Elliptic Cohomology

All four of these closely correspond to aspects of higher-dimensional CS-type field theory and its holographically related WZW-type field theory. As an important example, the objective is to work out the implications for the 7d/6d field theory following [S4].

B.1. Background

The traditional phenomena whose higher-dimensional generalization I propose to investigate via the new tool of local cohesive pre-QFT are the following.

Modular Functors. In mathematical *geometric quantization* (see [10] for a modern review) of topological field theory, the quantization step is essentially the process of attaching analogs of *theta functions* (i.e. modular and automorphic forms [47]) and *generalized theta functions* (conformal blocks) [7] to given geometric data over a codimension-one manifold Σ (a review is in [57]). Schematically, these are assignments $(z, \tau) \mapsto \theta(z|\tau)$ depending on a point z in the phase space of the CS theory—which is a flat connection, hence a *local system* on Σ—and on a point τ in the moduli space of *polarizing* geometric structure (typically: complex structure) on Σ. From the point of view of CS theory, such a theta function is a quantum *wavefunction* $\theta(z|\tau) = \Psi_\tau(z)$ with respect to the splitting of phase space into *canonical coordinates* and *canonical momenta* that is encoded by the polarization induced by τ [60,61]. The space of all theta functions from this point of view is the space of quantum states of CS theory over Σ. On the other hand, from view point of CFT, the theta function is the partition function $\theta(z|\tau) = \mathrm{Tr}(\exp(-\tau H_z))$ at the *Schwinger parameter* τ and for the background source field z [1]. From this perspective, the space of all such functions is called the space of *conformal blocks*. The collection of all these generalized theta functions in their dependence on τ is encoded by the *modular functor* [54]. This is the pivotal structure for all mathematical constructions of 3d CS theory (reviews include [56]) and thus, via holography, of 2d WZW theory [24].

$$\underbrace{L \mapsto \left(\underbrace{\Sigma \mapsto \left(\underbrace{z \mapsto \left(\underbrace{\tau \mapsto \theta(z|\tau)}_{\text{theta function}} \;\in\; \underbrace{\Theta_{z,\tau}}_{\text{theta line}} \right)}_{\text{Jacobi form}} \right)}_{\text{modular functor}} \right)}_{\text{CS theory in codimension 1}}$$

L	level / CS Lagrangian
Σ	manifold of codimension 1
z	flat G-connection/local systems on Σ
τ	geometric modulus on Σ

In conclusion, theta functions constitute a crossroad where the most delicate phenomena of modern quantum field and string theory touch deep mathematics in its purest arithmetic form.

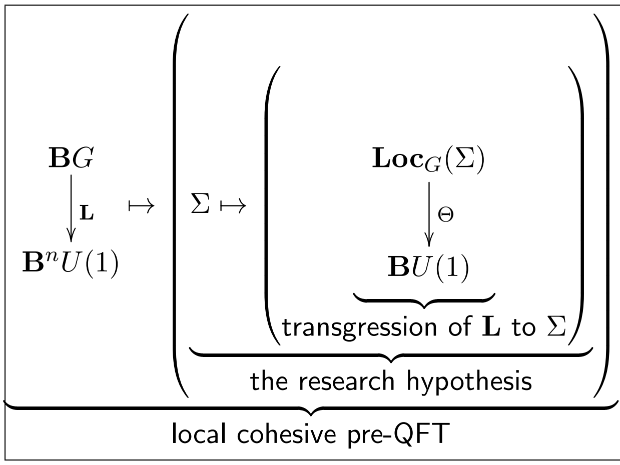
Relation with the Langlands Correspondence. Indeed, the Mellin transform of a theta function with respect to the Schwinger parameter, is its *zeta function* (*L-function*) $\zeta_z(s) := \int_0^\infty \tau^{s-1}\,\theta(z|\tau)\,d\tau$ defined as indicated where the integrand and its integral exist and analytically continued from there to a meromorphic function on the complex plane. When Σ is a torus, this is what in physics is called the *zeta-regularized one-loop vacuum amplitude* [55,13].

This assignment of zeta functions (L-functions) ζ_z to local systems z in the quantization of topological/conformal field theory is curiously reminiscent of the Langlands program in number theory (a good review is [26]): the main conjecture here states that there is a natural (Langlands: *functorial*) assignment of L-functions to Galois representations induced via associated automorphic forms. Noticing that Galois representations are the arithmetic incarnation of local systems and flat connections, they play the role of the z in the above story. Some aspects of this curious analogy of the Langlands correspondence with structures in CFT have been highlighted in [20]. However, the alternative proposal [38] for a relation of geometric Langlands to physics arguably had the effect of shifting attention away from this analogy. It seems worthwhile to reconsider the analogy with CFT and to consider it in higher-dimensional generalization following the cobordism hypothesis. In particular, it is compelling to study the higher incarnation of these phenomena for the case of 7d CS theory and 6d CFT.

Relation with Topological Modular Forms. When Σ is a marked torus, then a choice of complex structure makes it a complex elliptic curve. Passing from complex to arithmetic geometry yields the classical moduli stack of elliptic curves. Passing still further to its homotopy theoretic (*derived*) refinement [40] yields sheaves of elliptic spectra which constitute a homotopy theoretic refinement of the classical modular forms [32]. In a stupendous piece of work [3,2], it was shown that these *topological modular forms* canonically know about the partition function of the heterotic 2d CFT, namely that their cohomological orientation is a refinement of the Witten genus. This strongly indicates that any mathematical theory capturing the full depth of topological/conformal field theory needs to be rooted in geometric homotopy theory.

Relation with Equivariant Elliptic Cohomology. Indeed, organizing these elliptic spectra further into *equivariant elliptic cohomology* depending on a choice of gauge group [40] yields a homotopy-theoretic refinement of the modular functor assigning spaces of theta functions. This noteworthy fact is explicitly the content of [40] but has perhaps not found due appreciation yet. Moreover, [40] hints at a *2-equivariant* refinement of equivariant elliptic cohomology, where the higher theta line bundle appears naturally as the transgression of a fully local CS Lagrangian L to the moduli space of local systems on Σ.

This is directly suggestive of the research proposed here: in the context of higher differential geometry, I have studied in some detail [S6,S2,S7] and then put into the context of the cobordism hypothesis [S25] (based on discussion that recently led to [19]) a detailed incarnation of the idea of obtaining the theta line via transgression of a fully localized CS Lagrangian. Here, the observation is, first (see Track Record I.3)) that a CS level L may naturally be refined to a *higher line bundle* (*higher gerbe*) $\mathbf{L}$ on the universal moduli stack of CS-gauge fields and, second, that this defines a local (in the sense of the cobordism hypothesis) pre-QFT which automatically sends a closed manifold Σ of codimension-one to the transgression which is the theta line bundle over the moduli stack of local systems on Σ. Schematically, it means that given a fully localized CS Lagrangian $\mathbf{L}$ in the guise of a higher line bundle on a higher moduli stack, then the proof of the cobordism hypothesis induces a natural assignment of the form:



$\mathbf{L}$	localized higher CS-line bundle
$\mathbf{Loc}_G(\Sigma)$	higher moduli stack of local systems
Θ	theta line bundle

The point is that, on the one hand, for suitable choices of G and $\mathbf{L}$ this assignment does reproduce the traditional modular functor of 3d CS/2d WZW theory, while on the other hand this is a special case of something exceedingly more general: this naturally yields higher modular functors—as recently also considered in [19]—for higher-dimensional CS theories involving higher gauge fields, and does incarnate in a range of different flavors of geometry. Indeed, all we need is that $\mathbf{L}$ lives in *cohesive homotopy theory* [S25] which includes higher differential geometry [S14,S13] but also supergeometry [S8] and, crucially, also derived arithmetic geometry [S23]. Notably, the 7d CS theories for which I have constructed fully local $\mathbf{L}$ in [S4] fit into this construction and induce this way modular functors in dimension 6 which ought to encode partition functions of the mysterious 6d CFT.

B.2. Research Methodology

Objective 1 – Higher Modular Functors. The traditional modular functor of 2d CFT has different incarnations depending on how the conformal anomaly cancellation is taken into account [54,31].

Methodology: The proof of the cobordism hypothesis, however, yields a coherent picture of modular functors for higher-dimensional field theory [19]. Combined with the local cohesive construction of higher CS functionals from Part I, this yields a systematic procedure for constructing and studying higher modular functors, in particular for the case of 7d CS theory. It is underappreciated that the only actual definition of the 6d super-CFT is as the holographic dual of a 7d theory, which in a suitable limit is a nonabelian 7d CS-type theory. In the simple case of an abelian theory this has in fact been established in [61]. In [S4,S5], the full nonabelian 7d CS theory has been constructed in the fully local form needed as input for the topological field theory construction via the cobordism hypothesis.

Milestones, Expected Outcomes, and Time Frame: Milestones include the proof of conjecture 1.4 in [29]; computation via this (proven) conjecture of the homotopy fixed point structure on the local CS Lagrangians à la [42]; relating the resulting structure to the traditional Atiyah 2-framing structure [5] and to the traditional division-by-half of the Lagrangians on Spin-cobordisms [36]; computation of the 6d modular functor induced this way from the 7d theory of [S4]; relating this to the physics of the 6d super-CFT.

An expected outcome is a systematic understanding of modularity and anomaly cancellation in higher-dimenesional CS theory from the fundamental perspective of the proof of the cobordism hypothesis applied to local cohesive pre-QFT. The expected time frame for the basic mechanisms and examples is two years, a more in-depth study of the examples will take longer.

Objective 2 – Higher Topological Modular Forms. When Σ is a torus, the classical theta functions on the moduli of complex structures on Σ famously are examples of modular forms, which are instances of automorphic forms. By [27,40], these modular forms have a homotopy theoretic refinement to *topological modular forms* obtained by refining the formal group underlying elliptic curves to complex-oriented ring spectra, a construction that turns out to reflect deep aspects of 2d CFT [3,2]. This has motivated mathematicians to consider more general *topological automorphic forms* induced from 1d formal group laws split off from moduli on higher-dimensional Σ.

Methodology: The physics story of the higher CS/WZW-type duality just reviewed predicts a more accurate picture: the abelian varieties that matter are the higher (Griffith/Weil-)Jacobians $\mathbf{Loc}_{(\mathbf{B}^{2k+1}U(1))}(\Sigma)$ induced by $(4k+2)$-dimensional Σ (instead of Σ itself) and complex (or, more generally, arithmetic) moduli are supposed to be induced from the complex/arithmetic structures on Σ, see [S20] for review and further pointers. (This distinction is invisible in low dimension due to the coincidence that for Σ a torus/elliptic curve, then it so happens to be canonically equivalent to its own Jacobian!) Accordingly the formal group associated with an elliptic curve, whose identification leads to the concept of topological modular forms, is more properly thought of as the deformation theory of its moduli of line bundles, and so the proper higher analogs of that are the formal groups of deformations of higher Jacobians.

These formal groups appeared way back in [4], or, rather [4] considered formal groups of deformations of $(2k+1)$-line bundles on Σ and $(2k+1)$-connections on Σ. An old theorem of Deligne (reviewed in [S20]) says that to describe the higher Jacobians properly one needs a case in between these two i.e. $(2k+1)$-bundles with $k+1$-form connection. The higher topological modular forms associated with these are to be studied.

Milestones, Expected Outcomes, and Time Frame: The milestones include the identification of the correct formal groups and computation of their height; construction of associated complex-oriented spectra, hence of the generalized cohomology theory replacing elliptic cohomology in higher dimension (envisioned as *Calabi–Yau cohomology* in [53]); and in particular doing this in the special case of 6d Σ.

An expected outcome is a first picture of the correct nature of the cohomological lift of the 6d modular forms in higher analogy of the elliptic cohomology and topological modular forms in 2d. The expected time frame for first results is one year, an in-depth study will take longer.

Objective 3 – Higher Langlands Correspondence. While geometric Langlands duality focuses on the duality aspect of the theory, the number theoretic Langlands correspondence is primarily more directed: Langlands' crucial *Conjecture 3* (e.g. [26]) states that to a suitable Galois representation is naturally (Langlands: *functorially*) associated a certain zeta function, and that this may be expressed in terms of an automorphic form in correspondence with the Galois representation. These zeta functions appear as Mellin transforms of theta functions (just as zeta functions arise as 1-loop vacuum amplitudes in QFT). Keeping in mind that the geometric interpretation of a Galois representation is a flat connection, this means that Langlands' Conjecture 3 asserts the natural assignment of a theta function to a flat connection.

Methodology: It is striking that structurally this is precisely what the Research Hypothesis produces, with its theta functions naturally parametrized over the moduli $\mathbf{Loc}_G(\Sigma)$ of flat G-connections. But the claim there states a considerable generalization of this assignment: on the one hand, Σ may be a higher-dimensional curve but on the other hand the geometric context may be any suitably *cohesive* context. It remains to be further seen how this relates to established number theoretic Langlands, the result in [S23] already shows that there is a realization of cohesive homotopy theory (*arithmetic cohesion*) which does reproduce structures that are key to number theoretic Langlands.
Milestones, Expected Outcomes, and Time Frame: Milestones include a deeper analysis of arithmetic cohesion [S23] and discovery of potential variants; arithmetic realization of the local CS Lagrangians in algebraic K-theory following [15], construction of the zeta functions obtained via Mellin transform from the canonical theta bundles as indicated above at Background; relation to zeta-regularized vacuum amplitudes of the corresponding field theories, notably of the 6d theory.

An expected outcome are results on the structure of a higher analog geometric Langlands correspondence in higher generalization of the CFT-perspective of [20]. The expected time frame for first results is three years, an in-depth understanding is open-ended.

Objective 4 – Higher Equivariant Elliptic Cohomology. Arguably the most refined cohomological incarnation of the traditional theory of the 2d WZW model is given by the *2-equivariant elliptic cohomology* indicated in [40]. In outline[1] this is just the kind of construction that takes the 3d higher CS bundle to the theta line bundles that it induces by transgression, as indicated in Background. Of course the key aspect of [40] is to formulate this in *derived arithmetic geometry*, but this is just the context of Objective 3 above.

Methodology: Conceptually, the sketch of 2-equivariant elliptic cohomology in [40] is strikingly analogous to my Research Hypothesis . Thus, it seems plausible that the research proposed here leads to higher analogs of 2-equivariant elliptic cohomology. This is ambitious, but worth thinking about.

Milestones, Expected Outcomes, and Time Frame: Milestones include a formal identification of the trans-

[1] See http://ncatlab.org/nlab/show/equivariant+elliptic+cohomology

gression construction described in Background with the kind of higher equivariance in [42]; formulating the higher dimensional analog for the 7d CS Lagrangian of [S4]. The expected outcome are first insights into the mechanism of n-equivariance as a systematic phenomenon induced via the proof of the cobordism hypothesis applied to local cohesive pre-quantum field theory. The expected time frame is one year for first results, an in-depth understanding is open-ended.

[S1] D Fiorenza, C Rogers, U Schreiber, Int J Geom Methods Mod Phys, 10 (2013) 1250078

[S2] D Fiorenza, C Rogers, U Schreiber, arXiv:1304.0236

[S3] D Fiorenza, C Rogers, U Schreiber, Homology, Homotopy and Applications, Vol 16 No 2 (2014), 107-142

[S4] D Fiorenza, H Sati, U Schreiber, Adv Theor and Math Phys, Vol 18 No 2 (2014)

[S5] D Fiorenza, H Sati, U Schreiber, Comm Math Phys (2014), arXiv:1202.2455

[S6] D Fiorenza, H Sati, U Schreiber, J Geom and Phys, Vol 74 (2013) 130-163

[S7] D Fiorenza, H Sati, U Schreiber, in D Calaque et al (eds) Springer (2014), arXiv:1301.2580

[S8] D Fiorenza, H Sati, U Schreiber Int J Geom Meth Mod Phys (2014), arXiv:1308.5264

[S9] D Fiorenza U Schreiber, J Stasheff, Adv in Theor and Math Phys, Vol 16 No 1 (2012), 149-250

[S10] T Nikolaus, U Schreiber, D Stevenson, J Hom Rel Struct (2014) I, II, arXiv:1207.0248, arXiv:1207.0249

[S11] H Sati, U Schreiber, arXiv:1109.0955, in H Sati, U Schreiber (eds), AMS Proc Symp, Vol 83 (2011)

[S12] H Sati, U Schreiber, J Stasheff, Birkhäuser, Basel (2009), arXiv:0801.3480

[S13] H Sati, U Schreiber, J Stasheff, Comm Math Phys, Vol 315 (2012)

[S14] U Schreiber, Oberwolfach report No 28 (2009)

[S15] U Schreiber, lectures at *ESI Program on K-Theory and Quantum Fields*, Vienna (2012), (notes)

[S16] U Schreiber, lectures at *Workshop on Topological Aspects of Quantum Field Theories*, Singapore (2013)

[S17] U Schreiber, lectures at *Chern–Simons Theory: Geometry, Topology and Physics*, Pittsburgh (2013), (notes)

[S18] U Schreiber, talk at *Eighth Scottish Category Theory Seminar*, Edinburgh (2013), (notes)

[S19] U Schreiber, talk at *Formalization of Mathematics*, IHP Paris (2014), (notes)

[S20] U Schreiber, talk at *Higher Geometric Structures along the Lower Rhine*, Nijmegen (June 2014), (notes)

[S21] U Schreiber, talk at *Symmetries and Correspondences in Number Theory, Geometry, Algebra, Physics: Intra-Disciplinary Trends*, Oxford (2014), (notes)

[S22] U Schreiber, lectures *Workshop on Higher Gauge Theory and Higher Quantization*, Edinburgh (2014), (notes)

[S23] U Schreiber, talk at *Workshop on Differential Cohomologies*, New York (2014), (notes)

[S24] U Schreiber, talk at Operator and Geometric Analysis on Quantum Theory, Trento (2014), (notes)

[S25] U Schreiber, Habilitation thesis, Hamburg (2011), arXiv:1310.7930 (expanded pdf with errata)

[S26] U Schreiber, Proceedings of *Conference on Type Theory, Homotopy Theory, and Univalent Foundations*, (2013)

[S27] U Schreiber, arXiv:1402.7041

[S28] U Schreiber, M Shulman, in R Duncan (ed), *Proceedings 9th QPL*, Brussels (2012)

[1] L Alvarez-Gaumé, G Moore, C Vafa, Commun Math Phys, Vol 106 No 1 (1986), 1-4
[2] M Ando, M Hopkins, C Rezk (2010) (pdf)
[3] M Ando, M Hopkins, N Strickland, Invent Math, 146 (2001) 595-687
[4] M Artin, B Mazur, Annales scientifiques de École Normale Supérieure, Sér. 4, Col 10 No 1 (1977), 87-131
[5] M Atiyah, Topology, Vol 29 (1990) 1-7
[6] S Awodey, M Shulman, V Voevodsky et al, HoTT MURI grant (proposal pdf)
[7] A Beauville, Y Laszlo, Comm Math Phys Vol 164 No 2 (1994), 385-419
[8] A Berenbeim, MSc thesis at Univ Waterloo (in preparation, research diary at topostheorist.wordpress.com)
[9] J Bergner, arXiv:1011.0110 in H Sati, U Schreiber (eds), AMS Proc Symp Pure Math Vol 83 (2011)
[10] S Bongers, MSc thesis, Utrecht (2013), ncatlab.org/schreiber/show/master+thesis+Bongers
[11] U Bunke, T Nikolaus, arXiv:1406.3231
[12] U Bunke, T Nikolaus, M Völkl, J Hom Rel Struct, arXiv:1311.3188
[13] A Bytsenko, V Moretti, et al *Analytic Aspects of Quantum Fields*, World Scientific (2003)
[14] A Cattaneo, G Felder, Commun Math Phys, Vol 212 (2000), 591-611
[15] P Deligne, JL Brylinski, Publications Mathematiques de IIHS 2001 (web)
[16] J Distler, D Freed, G Moore, arXiv:0906.0795, in H Sati, U Schreiber (eds), AMS Proc, Vol 83 (2011)
[17] C Douglas, C Schommer-Pries, N Snyder, arXiv:1312.7188
[18] D Freed, Int Press, Somerville, MA (2000) arXiv:hep-th/0011220
[19] D Fiorenza, A Valentino, arXiv:1409.5723
[20] E Frenkel, in *Frontiers in number theory, physics, and geometry II*, Springer (2007), 387-533
[21] D Freed, talk at *ASPECTS of Topology*, Oxford (2012) (notes)
[22] D Freed, M Hopkins, Bull. Amer. Math. Soc. 50 (2013) 431-468
[23] D Freed, M Hopkins, J Lurie, C Teleman, in PR Kotiuga (ed), AMS (2010), arXiv:0905.0731
[24] I Runkel, J Fjelstad, J Fuchs, C Schweigert, Contemp. Math. 431:225-248 (2007)
[25] A García-Raboso, PhD thesis (2014) (pdf)
[26] S Gelbart, Bull Amer Math Soc, Vol 10 No 2 (1984), 177-219
[27] P Goerss, M Hopkins, in Lond Math Soc Lect Notes, Camb Univ Press, Vol 315 (2004), 151-200
[28] S Gukov, E Martinec, G Moore, A Strominger, in M Shifman et al (ed), 2004, arXiv:hep-th/0403225
[29] R Haugseng, arXiv:1409.0837
[30] A Henriques, Compositio Mathematica, Vol 144 No 4 (2008), 1017-1045
[31] N Hitchin, Comm Math Phys, Vol 131 No 2 (1990), 347-380
[32] M Hopkins, Proceedings of the ICM, Beijing Vol 1 (2002), 283-309
[33] M Hopkins, J Lurie, `http://www.math.harvard.edu/~lurie/papers/Ambidexterity.pdf`
[34] M Hopkins, I Singer, J Diff Geom Vol 70 No 3 (2005), 329-452
[35] J Huerta, PhD thesis, 2011, arXiv:1106.3385
[36] J Jenquin, arXiv:math/0504524
[37] A Kapustin, N Saulina, arXiv:1012.0911, in H Sati, U Schreiber (eds), AMS Proc Symp, Vol 83 (2011)
[38] A Kapustin, E Witten, Comm in Num Theor and Phys, Vol 1 No 1 (2007), 1-236
[39] W Lawvere, Theor Appl of Categ, Vol 19 No 3 (2007), 41-49
[40] J Lurie, in Algebraic Topology Abel Symposia (Vol 4 (2009), 219-277)
[41] J Lurie, Princeton Univ Press, (2009), arXiv:math.CT/0608040
[42] J Lurie, Current Developments in Mathematics Vol 2008, (2009), 129-280
[43] J Maldacena, Adv Theor Math Phys, Vol 2 (1998), 231-252
[44] G Moore, Felix Klein lectures Bonn (2012), (notes)
[45] G Moore, talk at Strings 2014, (notes)
[46] G Moore, E Witten, JHEP 0005 (2000) 032 arXiv:hep-th/9912279
[47] D Mumford, Tata Lectures on Theta, Birkhäuser, 1983
[48] S Müller-Stach, C Peters, V Srinivas, J Math Pures et Appl, Vol 98 No 5 (2012), 542-573
[49] J Nuiten, MSc thesis, Utrecht, 2013, ncatlab.org/schreiber/show/master+thesis+Nuiten
[50] T Pantev, B Toen, M Vaquie, G Vezzosi, Publ Math IHES, arXiv:1111.3209
[51] C Rezk, www.math.uiuc.edu/∼rezk/global-cohesion.pdf
[52] S Saito, arXiv:1405.0923
[53] H Sati, JHEP 0603 (2006) 096 arXiv:hep-th/0511087
[54] G Segal, in U Tillmann (ed), Lond Math Soc Lect Notes, Camb Univ Press, Vol 308 (2004), 421-577
[55] E Speer, Comm Math Phys, Vol 23 (1971), 23-36
[56] S Stirling, arXiv:0807.2857
[57] A Tyurin, AMS 2003 arXiv:math/0210466
[58] Univalent Foundations Project, Institute for Advaced Study, Princeton 2013, homotopytypetheory.org/book/
[59] F Wagemann, C Wockel Trans Amer Math Soc, 2014 arXiv:1110.3304
[60] E Witten, Commun Math Phys, Vol 121 No 3 (1989), 351-399
[61] E Witten, J Geom Phys, Vol 22 (1997), 103-133 arXiv:hep-th/9610234
[62] E Witten, JHEP 9812 (1998), 012 arXiv:hep-th/9812012
[63] E Witten, JHEP 0005 (2000) 03, arXiv:hep-th/9912086
[64] E Witten, in U Tillmann (ed), Lond Math Soc Lect Notes, (2004), arXiv.0712.0157
[65] E Witten, in P Kotiuga (ed), AMS (2010), arXiv:0905.2720