

*Higher theta functions
and
higher CS/WZW holography*

Proposing an
**interdisciplinary mathematics research
project**
based at Surrey, UK

Urs Schreiber

Career path

PhD in Essen-Duisburg (2005)

Postdocs in Hamburg, Utrecht, Nijmegen

Research fellowships at HIM Bonn and Notre Dame, USA

Habilitation in Hamburg (2011)

Visiting professorship in Prague (present)

[need long-term perspective right now](#)

Publications

29+ research articles

13 international coauthors

edited survey for Amer. Math Society (2011)

preprint monograph

Invited conference talks

plenty, e.g.:

- 2014 Oxford, Paris (IHP, Diderot),
New York (CUNY), Vienna (ESI), ...
- 2015 Göttingen, Prague,
Porto (Amer. + Eur. Math Society meeting),
Hamburg (Germ. Math Society meeting, 2 talks), ...

Invited lecture series

- 2013 Singapore (NUS), Bonn (MPI), Pittsburgh
- 2014 Edinburgh (Heriot-Watt), Paris (Diderot)
- 2015 Prague, Pittsburgh

Impact and Leadership

'08: created research website:



communication platform

recording research level mathematics

focus on topics related to this proposal

'10: cited widely by scholarly sites:

Wikipedia, PlanetMath, MathOverflow, PhysicsOverflow,...

'11: open publishing experiment: *Publications of the nLab*

needs dedicated staff and funding

'15: migrated to server at Carnegie-Mellon

funded by DoD MURI grant '14

Background I: **The rise of homotopy theory**

- '70s: Grothendieck (Fields medal '66) envisions:
*“algebraic geometry and number theory
will reveal deep secrets via **homotopy theory**”*
- '95: Baez (UCR), my de-facto PhD advisor, conjectures
cobordism hypothesis:
“homotopy theory governs quantum field theory”
- '09: Lurie (Harvard) finds **proof**, realizes Grothendieck's dream
Breakthrough Prize '14, MacArthur Award '14
- '13: Awodey (CMU), Voevodsky (IAS, Fields medal '02) find:
*“homotopy theory serves as new **foundations**”*
DoD MURI Grant '14

At this point my proposal comes in:

Novel method introduced in my Habilitation (2011):

cohesive homotopy theory: “inter-geometric” $\left\{ \begin{array}{l} \text{differential} \\ \text{arithmetic} \\ \dots \end{array} \right.$

remarkable **consequence**:

Claim Θ (Research Hypothesis):
cohesive homotopy theory + cobordism hypothesis
 \Rightarrow *higher Theta functions*

proven (with D. Fiorenza) but yet unpublished...

...research grant for exploiting this gold mine

Background II: **Theta functions**

19c: hallmark of **number theory**
give famous zeta functions

'80s: appearance in **quantum field theory**
quantum correlators, vacuum amplitudes

At the heart of two major international research programs:

arithmetic geom.: **Langlands program**

Galois representations \mapsto zeta functions

\updownarrow inter-geometric analogy

differential geom.: **quantization** of **3dCS/2dWZW** theory
gauge fields \mapsto vacuum amplitudes

Background III: **Higher CS/WZW holography**

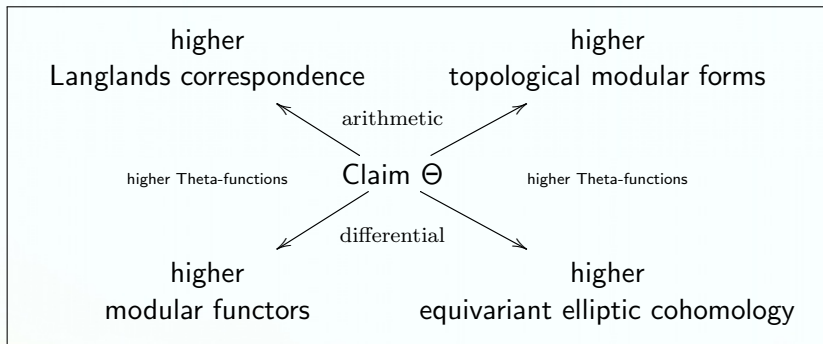
- '90: Witten receives Fields medal
for **new mathematics suggested from quantum field theory**
- '97: Maldacena finds *holographic principle*
becomes the **most cited** topic in the new millenium
- '00: Witten relates it to *higher* dimensional CS/WZW:

	holography		
3dCS	\leftrightarrow	2dWZW	“string”
7dCS	\leftrightarrow	6dWZW	“M5-brane”
11dCS	\leftrightarrow	10dWZW	“RR-fields”

The mathematics for the latter is barely existent...

...but **Claim Θ** provides just that: \longrightarrow

Objectives – *Claim* $\Theta \Rightarrow$ novel unified handle on 4 nascent topics:



cutting edge, highly interdisciplinary, broad impact

UK hosts all relevant expertise, esp. at Surrey and vicinity
would make UK international focus point for these new developments

Academic Beneficiaries

plenty, due to the **novel level of interdisciplinarity**. E.g.:

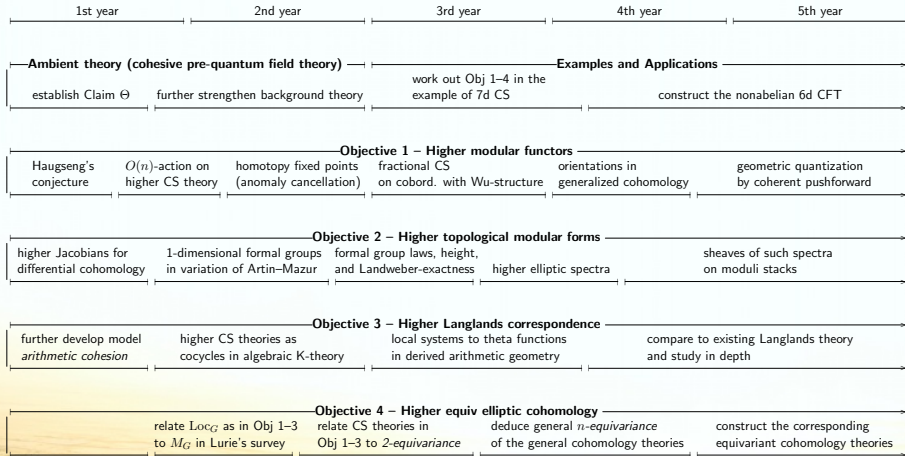
UK

J Gutowski, J McOrist, S Pasquetti, A Torrielli, M Wolf (Surrey)
O Ben-Bassat, N Hitchin, D Joyce, Y Kremnitzer, G Segal (Oxford)
M Atiyah (Edinburgh), K Buzzard (King's), M Green (Cambridge)
T Altenkirch, I Fesenko, S Oblezin (Nottingham),
A Schenkel, C Sämann, R Szabo (Heriot-Watt),
E Cheng, N Gurski, K Mackenzie, N Strickland (Sheffield),
N Gambino (Leeds), D Corfield (Kent), J Ladyman (Bristol), ...

international

D Freed (Austin), J Lurie (Harvard), M Ando (Illinois),
N Ganter (Melbourne), U Bunke (Regensburg),
A Henriques (Utrecht), G Moore (Rutgers), ...

Workplan – detailed strategy based on my previous work:



plenty of strong sub-goals \Rightarrow much contingency

Ambitious but Viable

have been developing **much theoretical infrastructure**

in my monograph and lecture notes

⇒ preliminary results, immediate starting point of project

⇒ concrete goals for the PhD student

First results since submission of proposal:

Fiorenza, Schreiber, Valentino:

Extensions of mapping class groups from characteristic classes

arXiv:1503.00888

Fiorenza, Sati, Schreiber:

The WZW term of the M5-brane and differential cohomotopy

arXiv:1506.07557

First Impact – invited to contribute to book collection:

Baez, Cartier, Cisinski, Joyal ,
Kapranov, Kontsevich, Manin, Marcolli,
Penrose, Schreiber, Simpson et al.:

New Spaces for Mathematics and Physics

IHP Paris (end of 2015)

funded by European Research Council

Expert referee judgement

“ambition of the proposed methodology is outstanding”

“scope of the areas is remarkable”

“can truly be called mathematics of the 21st century”

“wish there were more EPSRC proposals of this type”

Claim Θ

cohesive homotopy theory:	$Z(*) := \mathbf{L}_{\text{CS}}$
cobordism hypothesis:	$Z(*) \Rightarrow Z(\Sigma)$
claim:	$Z(\Sigma) = \Theta$

$$\underbrace{\begin{array}{c} \mathbf{BG} \\ \downarrow \mathbf{L}_{\text{CS}} \\ \mathbf{B}^n U(1) \end{array}}_{Z(*)} \mapsto \left(\Sigma \mapsto \underbrace{\left(\begin{array}{c} \mathbf{Loc}_G(\Sigma) \\ \downarrow \Theta \\ \mathbf{BU}(1) \end{array} \right)}_{Z(\Sigma)} \right)$$

higher dim
Chern-Simons
Lagrangian

higher Theta line